Combining Exploratory Learning With Structured Practice to Foster Conceptual and Procedural Fractions Knowledge

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Abstract: Robust domain knowledge consists of conceptual and procedural knowledge. The two types of knowledge develop together, but are fostered by different learning tasks. Exploratory tasks enable students to manipulate representations and discover the underlying concepts. Structured tasks let students practice problem-solving procedures step-by-step. Educational technology has mostly relied on providing only either task type, with a majority of learning environments focusing on structured tasks. We investigated in two quasi-experimental studies with 8-10 years old students from UK (N = 121) and 10-12 years old students from Germany (N = 151) whether a combination of both task types fosters robust knowledge more than structured tasks alone. Results confirmed this hypothesis and indicate that students learning with a combination of tasks gained more conceptual knowledge and equal procedural knowledge compared to students learning with structured tasks only. The results illustrate the efficacy of combining both task types for fostering robust fractions knowledge.

Keywords: conceptual knowledge, procedural knowledge, exploratory learning environments, intelligent tutoring systems, mathematics education

Introduction

Robust domain knowledge consists of two types of knowledge, namely conceptual and procedural knowledge (Anderson, 1987; Rittle-Johnson, Siegler, & Alibali, 2001). Previous research has mostly focused on fostering either type of knowledge. This work explores how a combination of different types of instructional support provided by educational technology can foster both conceptual and procedural fractions knowledge.

Conceptual knowledge can be defined as implicit or explicit understanding about underlying principles and structures of a domain (Rittle-Johnson & Alibali, 1999). The focus of this type of knowledge lies on understanding why, for example, different mathematical principles refer to each other and on making sense of these connections. Procedural knowledge can be defined as knowledge about and application of procedures (Rittle-Johnson & Alibali, 1999). Procedures are an action sequence of, for instance, mathematical problem-solving steps (Rittle-Johnson & Alibali, 1999). The main aspect of procedural knowledge is in knowing how to apply a rule in order to solve a problem. According to Anderson's ACT-R model (e.g., Anderson, 1982), procedural knowledge becomes implicit with increasing practice.

Both types of knowledge develop over the same period of time (Canobi, Reeve, & Pattison, 2003; LeFevre et al., 2006) and evolve in a relationship of mutual dependence (Rittle-Johnson & Koedinger, 2009; Rittle-Johnson et al., 2001). Conceptual and procedural knowledge develop iteratively "with increases in one type of knowledge leading to gains in the other type of knowledge, which trigger new increases in the first" (Rittle-Johnson et al., 2001, p. 347).

While the development of these types of knowledge thus coincides (Rittle-Johnson et al., 2001), learning activities are thought to differ in which type of knowledge they primarily foster (Koedinger, Corbett, & Perfetti, 2012). Exploratory learning activities provide space for students to discover the underlying (mathematical) principles by abstracting concrete information and constructing schemata, thus primarily fostering conceptual knowledge. Structured practice activities introduce problem-solving procedures step-by-step and offer repeated (structured) practice opportunities for acquiring and deepening of these procedures (Anderson, Boyle, Corbett, & Lewis, 1990), thus primarily fostering procedural knowledge.

Educational technology has mostly focused on supporting either one or the other type of learning activity. Exploratory learning environments (ELEs), often referred to as micro-worlds, allow students, for example, to manipulate representations (e.g., Mavrikis, Gutiérrez-Santos, Geraniou, & Noss, 2013) and crucially, to explore the mathematical relationships between and within the representations and their underlying concepts (Hoyles, 1993; Thompson, 1987). ELEs can support students in these activities by encouraging reflection and self-explanation (e.g., Mavrikis & Gutiérrez-Santos, 2009). Intelligent Tutoring Systems (ITS) guide students through solving problems step-by-step, and offer immediate feedback so that students can automatize the problem-solving procedure bit by bit (Koedinger & Corbett, 2006; VanLehn, 2006). This feedback is typically directed more at problem-solving rather than at understanding the underlying concepts.

Given that conceptual and procedural knowledge develop in mutual dependence (e.g., Rittle-Johnson et al., 2001), it is somewhat surprising that prior work in the learning sciences and educational technology has focused on fostering *either* procedural knowledge with structured tasks (within ITSs) *or* conceptual knowledge with exploratory tasks (within ELEs). One exception is Holmes (2013) who investigated a games-based environment that provided both opportunities for children to discover numeracy concepts for themselves, using authentic problems that could only be solved using mathematics, and opportunities for them to practice related procedures. While Holmes did not explicitly test this hypothesis, combining exploratory learning and structured practice tasks should be more effective for learning because it promotes the iterative development of conceptual and procedural knowledge: conceptual understanding that students can directly apply to problem-solving should in turn deepen the conceptual understanding.

Some indirect evidence for this *combination effect* comes, for example, from research on productive failure in physics education (Kapur, 2008) and iterative lesson sequencing in mathematics education (Rittle-Johnson & Koedinger, 2009). Kapur (2008) investigated whether attempting to solve ill-structured problems before solving well-structured problems can be more productive than solving problems in the reverse order. The problems shared similarities with exploratory learning activities and structured tasks as defined above: ill-structured problems required students to discover how to structure and solve them by abstracting concrete information, while the well-structured problems required the application of predictable rules and principles. Kapur found that students who had solved ill-structured tasks first outperformed their counterparts later on in solving both ill-structured and well-structured tasks. However, students worked on these problems collaboratively and without educational technology, or support. Furthermore, there was no control condition where students learned with only one type of task. Rittle-Johnson and Koedinger (2009) investigated iterative lesson sequencing (lessons that alternate in focusing concepts or procedures) with an ITS. They found that the iterative lesson sequence fostered procedural knowledge more than a concepts-before-procedures sequence and that there was no difference in conceptual knowledge. However, the lessons that focused on concepts were heavily structured and did not provide the affordances for discovery that ELEs offer. Taken these limitations in mind, these findings still suggest that a learning environment combining exploratory and structured tasks could foster conceptual and procedural knowledge, and more so than structured tasks alone which the majority of existing tutoring environments provide.

In summary, while there is theoretical ground and indirect empirical evidence for combining exploratory learning with structured practice to promote conceptual and procedural knowledge acquisition, this hypothesis has not yet been explicitly tested. We report on two studies which investigated this question in two countries (Germany and the UK), using a newly-developed learning platform for fractions learning: iTalk2Learn. It is the product of an interdisciplinary research project funded by the EC under the 7th FP (italk2learn.eu.). Fractions were chosen as the learning domain because this mathematical topic is particularly difficult and challenging for young students (Charalambous & Pitta-Pantazi, 2007), and because students' fractions ability is a predictor for future math performance (Siegler et al., 2012)

Methods

Experimental design

The two studies were part of a larger research design that also investigated the impact of using speech to adapt to learners' needs. This paper reports data from two experimental conditions:

(Full Platform)	The full iTalk2Learn platform incorporating exploratory learning and structured practice.
(No ELE)	The iTalk2Learn platform incorporating structured practice, but not exploratory learning.

Due to the readily observable differences in learning tasks between the conditions, it was not feasible to run multiple conditions in the same classroom. Therefore, the studies were run in a quasi-experimental design.

Participants

Participants in both countries were students who were just about to start or at the beginning of formal fractions instruction. Fractions are taught earlier in the curriculum in the UK than in Germany. Parental consent for their involvement in the study was obtained for all participating students.

Participants in the study in the UK were Year 4 and Year 5 primary school students aged between 8 and 10 years from three schools. The schools were from a rural, suburban, and inner-city area. Seven students did not complete the study. Students were roughly stratified, according to previous teacher assessments of the children's mathematical ability, in three groups per grade per school which were then randomly assigned to one of the conditions, resulting in the following distribution across conditions: $N_{\text{Full Platform}} = 61$ and $N_{\text{No ELE}} = 60$.

Participants in the study in Germany were fifth and sixth grade secondary school students aged between 10 and 12 years from four schools from suburban areas. Participating students could not be stratified due to timetable constraints of the participating schools, so students participated within their class, and classes within schools were randomly assigned to one of the conditions. Class sizes varied, and, due to a technical failure, data was lost for one class of 33 students assigned to the No ELE condition, resulting in the following distribution across conditions: $N_{\text{Full Platform}} = 100$, and $N_{\text{No ELE}} = 51$.

Dependent measures

Participants completed an online and an offline fractions test, a questionnaire on attitudes to learning, mathematics and fractions, a questionnaire on their experience using the platform, and questions on their experience of the task they had just completed. We recorded all participant interaction with the platform, including speech. For a subsample of participants, while they worked with the platform observers assessed their affect. This paper reports data from the online fractions test.

For the online fractions test, two isomorphic versions were designed. Students were randomly allocated one version at the first time of measurement and the other version at the second time of measurement. Two subscales with three items each were constructed to measure procedural knowledge (see questions 22, 24, and 25 in Figure 1) and conceptual knowledge (see questions 20, 21, and 23 in Figure 1). The students received one point for each correctly-answered item and consequently obtained two aggregated scores, one per subscale. Internal consistency for the procedural scores at pre-test was $\alpha_{UK} = .40$, $\alpha_{Germany} = .07$, and at post-test $\alpha_{UK} = .53$, $\alpha_{Germany} =.36$. Internal consistency for the conceptual scores at pre-test was $\alpha_{UK} = .40$, $\alpha_{Germany} = -.03$, and at post-test $\alpha_{UK} = .36$, $\alpha_{Germany} = -.06$. The low internal consistency is addressed in the discussion.

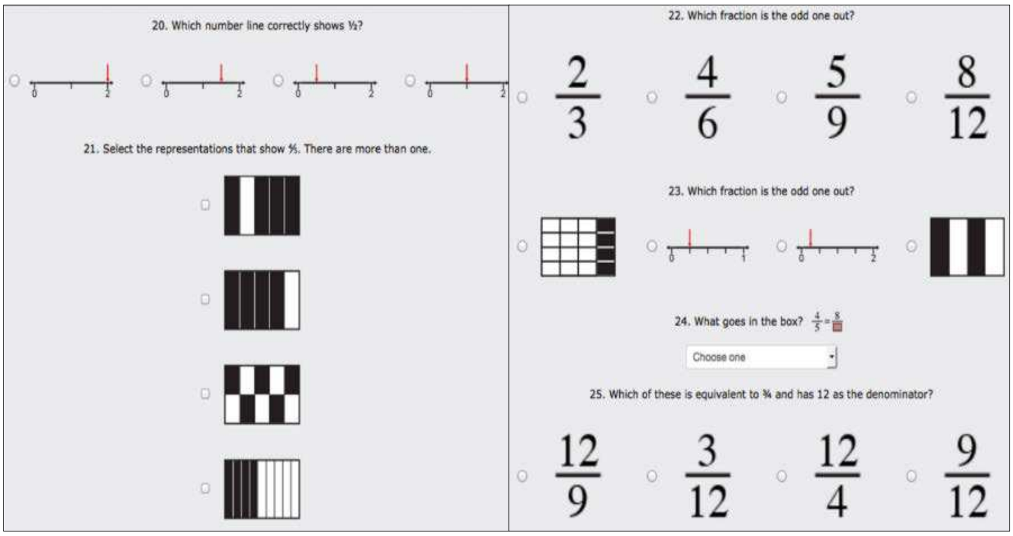


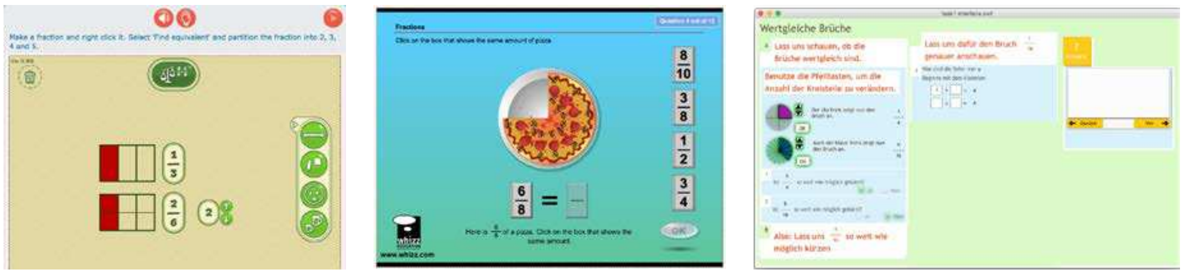
Figure 1. Online fractions test.

Procedure

Individual sessions were run with up to 15 students in the UK and up to 30 students in Germany. Each session lasted approximately 90 minutes including breaks. During the first 10 minutes, the students were introduced to the study and to the iTalk2learn platform with the components being introduced depending on the experimental condition. To ensure that the introduction was as standardized as possible, it was scripted and was delivered by the same researchers in each session. The students were then asked to complete several instruments. The online fractions test was presented following the questionnaire on attitudes to learning, mathematics and fractions, both together in one browser window. Students were given 10 minutes total for these two instruments. Students then worked with the iTalk2Learn platform for approximately 40 minutes. During this main experimental period, the researchers adopted an intervention protocol that specified the allowable interactions and prompts. In the last 30 minutes of the session, the students were asked to complete the final instruments. The online fractions test was presented following the user experience questionnaire, both together in one browser window. Students were given twenty minutes in total for these two instruments.

iTalk2Learn platform

The pedagogy of the iTalk2Learn platform is based on an intervention model for fostering robust knowledge described by Mazziotti et al. (2015). For the present studies, the model was instantiated for the topic of equivalent fractions. The platform combined an ELE delivering exploratory tasks, developed within the iTalk2Learn project, Fractions Lab (Hansen, Mavrikis, Holmes, & Geraniou, 2015; http://fractionslab.lkl.ac.uk), with one of two ITSs delivering structured tasks. In the UK, the ITS was a commercial system, Maths-Whizz (www.whizz.com); in Germany, it was an academic system, Fractions Tutor (e.g., Olsen, Belenky, Aleven, & Rummel, 2014; Rau, Aleven, & Rummel, 2013). The next section describes these learning environments and the tasks provided by them. Tasks were chosen by mathematics education experts based on an original coherent system for fractions learning that takes into account misconceptions and errors that are typical for learners at the beginning of formal fractions instruction (Hansen et al, 2014). Then, the adaptive support available to students is described. Finally, a section on the Student Needs Analysis (SNA) explains how tasks were sequenced within and switched between learning environments.



(a) (b) (c)

Figure 2. Exploratory Learning Environment (Fractions Lab; a), Structured Practice Environment used in the UK (Maths Whizz; b), and Structured Practice Environment used in Germany (Fractions Tutor; c).

Fractions Lab

Fractions Lab is an ELE that provides exploratory tasks that aim to help the student develop conceptual knowledge of fractions. In the Fractions Lab interface (see Figure 2a), a learning task is displayed at the top of the screen. Students can choose fraction representations (from the right-hand side menu) which they manipulate in order to solve the given task. For example, they can change the fraction's numerator or denominator, and find an equivalent fraction. An example task is shown in Figure 2a, which served both to introduce the student to available Fractions Lab functionality, and to introduce them to the idea and appearance of fraction equivalence with representations (Hansen et al., 2015).

Maths-Whizz

Maths-Whizz is a commercial system that provides structured practice content. This content is delivered in three stages: a teaching page which explains, procedurally, how to complete the following exercises successfully, interactive exercises with guided instruction and immediate feedback (see Figure 2b), and a short test. The exercises use a range of graphical representations such as circles, rectangles, number lines, liquid measures, symbols and sets of objects within contexts that the students may be familiar with.

Fractions Tutor

This web-based Cognitive Tutor for learning fractions (e.g., Olsen et al., 2014) enables students to solve fractions problems step-by-step, and receive immediate feedback or ask for on-demand hints. Content is presented on the same page and revealed step by step while students solve the problem (for an example, see Figure 2c). The exercises use a range of graphical representations such as circles, number lines, and symbols.

Adaptive support

While the students interacted with the ELE and with the ITS, they were given automatic task-independent support (TIS) and task-dependent support (TDS).

Within the ELE, the type of support provided (TIS or TDS) was based on a Bayesian Network which aims to change a negative affective state, for example frustration or boredom, into a positive affective state such as enjoyment by adapting the feedback to the student's affective state. Affective states were inferred from the student's speech and from interaction data, that is, whether or not feedback had been followed. TIS support included affect boosts (e.g., "Well done. You're working really hard!"), or talk-aloud prompts (e.g., "Please explain what are you doing."). TDS support included instructive feedback (e.g., "Use the comparison box to compare your fractions.", Holmes, Mavrikis, Hansen, & Grawemeyer, 2015), more open-ended feedback (e.g. "Good. What do you need to do now, to complete the fraction?"), reflective prompts (e.g., "What do you notice about the two fractions?"), affirmation prompts (e.g., "The way that you worked that out was excellent."), or task sequence prompts (e.g. "Are you sure that you have answered the task fully? Please read the task again."). The way how the feedback was provided to the student (high- or low- interruptive) was adapted according to their affective states (Grawemeyer, Holmes, Gutierrez-Santos, Hansen, Loibl & Mavrikis, 2015).

Within the ITS, TDS was provided based on students' performance and consisted of highlighting mistakes and providing problem-solving instruction. TIS was provided as described above, based on the Bayesian network and adapted to students affect states deduced from their speech.

SNA: Sequencing within and switching between learning environments

In the Full Platform condition, students began their iTalk2Learn session in the ELE. While the student was engaged with the ELE, the Student Needs Analysis (SNA) component drew on various inputs (e.g., screen/mouse action, speech) to determine whether the student was under-, over-, or appropriately challenged by the task and thus to identify the next task appropriate for them. After each second task completed by the student, the SNA switched to the alternative type of task (i.e. when they had completed two exploratory tasks, they were switched to the ITS, and vice versa). If the student was switched to the ELE, the level of challenge that they had experienced on the previous task was taken into account when calculating the next task. The first task in the ITS was mapped to the fine-grain goal of the completed task in the ELE (e.g., partition a fraction to find its equivalent). The next task in the ITS stayed within the same fine-grain goal but increased the level of challenge based on a sequence determined by math education experts. Students continued in this fashion, alternating between exploratory learning and structured practice every second task until the 40 minutes were concluded. In the No ELE condition, students worked on the ITS only and received tasks based on the same sequence used in the Full Platform condition.

Findings

Table 1 presents scores on the online fractions knowledge test for the conceptual and procedural subscales. There was a medium correlation between these subscales on the post-test, $r(151) = .25$ in Germany and $r(121) = .26$ in UK, both $p < .01$.

Table 1: Scores on online fractions knowledge test

			Pre-test		Post-test		Effect size	
Subscale	Country	Condition	*M*	*SD*	*M*	*SD*	*d*	95% CI
Conceptual	Germany	Full Platform	0.79	0.74	1.21	0.69	0.59	[0.30, 0.87]
		No ELE	0.73	0.63	0.53	0.64	-0.32	[-0.71, 0.07]
	UK	Full Platform	1.00	0.95	1.52	0.85	0.58	[0.22, 0.94]
		No ELE	0.88	0.92	0.70	0.77	-0.21	[-0.57, 0.15]
Procedural	Germany	Full Platform	0.95	0.80	1.42	0.94	0.54	[0.26, 0.82]
		No ELE	0.69	0.65	0.90	0.85	0.28	[-0.11, 0.67]
	UK	Full Platform	1.33	0.96	1.97	1.02	0.65	[0.28, 1.01]
		No ELE	1.47	1.02	1.87	1.07	0.38	[0.02, 0.74]

Note. Scores are summed across three items per subscale and can vary between 0 and 3.

Two-way 2 (condition: Full Platform or No ELE) x 2 (time of measurement: pre-test or post-test) multivariate ANOVAs with repeated measures on the time variable and conceptual and procedural subscale scores as the two dependent measures were conducted for each country separately. Using Pillai's trace, analyses showed significant effects of time of measurement for participants from both Germany, $V = .117$, $F(2,148) = 9.834$, $p < .001$, $\eta_p^2 = .117$, and UK, $V = .277$, $F(2,118) = 22.643$, $p < .001$, $\eta_p^2 = .277$. There were also significant effects of condition for participants from both Germany, $V = .132$, $F(2,148) = 11.274$, $p < .001$, $\eta_p^2 = .132$, and UK, $V = .133$, $F(2,118) = 9.025$, $p < .001$, $\eta_p^2 = .133$. Importantly, there were also significant interaction effects of time of measurement and condition for participants from both Germany, $V = .109$, $F(2,148) = 9.068$, $p < .001$, $\eta_p^2 = .109$, and UK, $V = .114$, $F(2,118) = 7.604$, $p < .001$, $\eta_p^2 = .114$. Results were similar in both countries: Students in both conditions showed learning gains, but these were stronger for the Full Platform condition. This interaction is now investigated further.

Follow-up univariate analyses showed significant effects of time of measurement on the procedural scores for participants from both Germany, $F(1,149) = 18.552$, $p < .001$, $\eta_p^2 = .111$ and UK, $F(1,119) = 16.337$, $p < .001$, $\eta_p^2 = .265$, but not on the conceptual scores for participants from neither Germany, $F(1,149) = 2.206$, $p > .05$, nor UK, $F(1,119) = 3.078$, $p > .05$. There were significant effects of condition on the procedural scores for participants from Germany, $F(1,149) = 10.618$, $p < .001$, $\eta_p^2 = .067$, but not from UK, $F(1,119) < 1$. For conceptual scores, there were significant effects of condition for participants from both Germany, $F(1,149) = 16.465$, $p < .001$, $\eta_p^2 = .100$, and UK, $F(1,119) = 13.999$, $p < .001$, $\eta_p^2 = .105$. Finally, there were no significant interaction effects of time and condition on the procedural scores for participants from neither Germany $F(1,149) = 2.552$, $p > .05$, nor UK, $F(1,119) = 2.279$, $p > .05$. But on the conceptual scores, there were significant interaction effects for participants from both Germany, $F(1,149) = 16.697$, $p < .001$, $\eta_p^2 = .101$, and UK, $F(1,119) = 13.245$, $p < .001$, $\eta_p^2 = .100$. Results were similar in both countries: Students in both conditions showed equal learning gains on procedural scores. The decrease of conceptual scores in the No ELE condition is not significant: the 95% confidence interval of the effect indicates that a small increase is similarly likely. Students in the Full Platform condition did show significant learning gains on conceptual scores.

Discussion

Robust knowledge consists of conceptual and procedural knowledge that need different types of instructional support. Yet, learning systems that have been developed for mathematics education are usually constrained either to exploratory tasks or to structured tasks and thus can promote learning only to a limited extent. We demonstrated in this paper our attempt to overcome this limitation by combining exploratory tasks from Fractions Lab, a newly-developed exploratory learning environment, and structured tasks from Maths-Whizz and Fractions Tutor, two proven intelligent tutoring systems.

Two studies provided clear evidence that the combination of exploratory tasks (to foster primarily conceptual knowledge) and structured tasks (to foster primarily procedural knowledge) in one learning environment promotes fractions knowledge more than state-of-the-art ITSs providing structured tasks only. More specifically, the combination of tasks led to stronger conceptual learning gains without hindering procedural learning. The latter is particularly remarkable given that learning time was split between exploratory and structured tasks in the Full Platform condition, and given that exploratory tasks primarily target conceptual knowledge. Interestingly, procedural learning gains were smaller in the No ELE than in the Full condition. Different from the contexts in which Maths-Whizz and Fractions Tutor are usually deployed, in the present studies students had very limited time to study very specific learning content. Moreover, participants had not worked with these learning environments before. Against this background, the clear learning gains observed in the Full Platform condition are even more impressive.

In spite of the overall results, there are some limitations worth discussing. The first limitation concerns the measurement of procedural versus conceptual knowledge. These constructs can hardly be measured independently (Schneider & Stern, 2010), but we emphasize that our measures are meant to primarily tap one versus the other type of knowledge. Moreover, in the German sample, the conceptual scores are not internally consistent which highlights the need to investigate their dimensional structure and further develop a valid measurement of procedural versus conceptual knowledge. That said, the clear result patterns overall does provide first evidence of an interesting effect. Following a multi-method approach, we have collected more data which, once analyzed, will shed further light on the validity of the results presented here. Another limitation is the short duration of the intervention. While conducting the studies in school classrooms increased external validity, it also placed constraints on the learning time available to us through the schools. An extended follow-up study may lead to more robust knowledge gains, evidenced by larger internal consistencies of measures, and retention and transfer effects.

Despite these limitations, and the differences between the two studies conducted in Germany and in the UK, the results were remarkably similar between the two countries. This speaks to the generalizability of our findings and the external validity of the combination effect. Finding evidence for the combination effect underlines the need to foster both types of knowledge jointly, as Rittle-Johnson et al. (2001) highlighted with their iterative model of knowledge development.

The effect of combining task types prompts a series of follow-up questions. One of these questions asks for the component that makes the combination effect effective. For example, is the order of exploratory followed by structured tasks essential for realizing the combination effect? Or could the order be reversed? We based the order of exploratory and structured tasks implemented in our studies on prior research that showed conceptual learning should be fostered first (e.g., Kapur, 2008). This principle was not only realized within the first two tasks, but formed one of the rules of our intervention model employed throughout learning with iTalk2Learn. The iTalk2Learn system now provides an additional, proven research context in which the generalizability of the prior research findings can be tested.

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