“Ohhh, Now I Can Do It!”: School-age Children’s Spontaneous Mathematical Sensemaking in Construction Play

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Abstract: This analysis joins together two lines of work: mathematical problem solving and children’s construction play as a resource for mathematics learning. Our study is motivated by two observations. First, play has characteristics reminiscent of professional mathematicians’ practice. Second, the child-centeredness of play points to possibilities for equitable mathematics instruction. We conceptualize *problem solving in construction play* as the process children engage in when they experience trouble accomplishing a goal and work to repair it. To examine this phenomenon, we used head-mounted GoPros to collect 348 point-of-view videos of children as they played in a mathematical playground, where “messing around” with designed objects provided them with opportunities to encounter mathematical concepts. Through a close examination of one case, we illustrate the *mathematical generativity* of play with the designed objects and argue that children’s construction play can support rich opportunities for children’s mathematical sensemaking.

Keywords: mathematics learning, out-of-school learning, play-based learning; mathematical sensemaking; interaction analysis

The heart of children’s play involves asking *what if* questions about the world. Similarly, the heart of mathematical activity involves asking *what if* questions about patterns in number, shape, and space. Thus, we posit that children’s play in mathematically rich contexts provides a productive avenue for them to conjecture and explore in disciplinarily authentic ways. Yet little is known about how school-age children might make sense of ideas in mathematically-rich play settings.

In the U.S., children’s opportunities to encounter mathematics are largely limited to classrooms. This is a significant shortcoming, since many children’s school mathematics experiences emphasize ritualized activity and pre-determined solutions (Stigler & Hiebert, 2009), often resulting in them disliking the discipline (Boaler & Greeno, 2000). This type of instruction is at odds with authentic disciplinary engagement (Engle & Conant, 2002) and mathematical practices like exploration, defining, and conjecture-testing (Lakatos, 1978).

This is a missed opportunity, since research has shown that self-selected and self-directed out-of-school learning helps students develop positive disciplinary identities and builds foundations for formal knowledge, even in technical fields like science, technology, and engineering (Quinn & Bell, 2013). Out-of-school settings facilitate children’s focus, persistence, joy, and pride in work (Petrich, Wilkinson, & Bevan, 2013). While out-of-school studies in museums and makerspaces have investigated students’ scientific, technological, and engineering activities, informal mathematics remains underexamined, offering little guidance for designing spaces that might support mathematical exploration.

We study out-of-school mathematics learning by investigating *mathematically generative play* in a mathematical playground called Math On-A-Stick (MOAS). To identify mathematically generative play, we look for moments when *what if* questions of play intersect with *what if* questions about disciplinary concepts. Specifically, we examine children’s *construction play* — play with materials that require building or designing. In construction play, children explore objects’ properties and see what they can do with them. In the process, they experiment and make tacit conjectures around patterns in number, shape, and space, which we call mathematical generativity. In addition to opportunities to do mathematics, mathematically generative play opens up possibilities for students to experience *disciplinary enjoyment* (Sengupta-Irving & Enyedy, 2015) of mathematics, potentially supporting their long-term identification with and persistence in mathematics.

Our work builds on previous studies of mathematical play, noting that they are primarily focused on young children whereas we focus on upper elementary and middle school age children. That research has shown that construction play is rich with opportunities for informal mathematics learning (Seo & Ginsburg, 2004), yet play of any kind is increasingly being pushed out of U.S. preschool, elementary, and middle schools due to increased emphasis on standardized tests and standardized curricula (Bodrova & Leong, 2003; Jerret, 2015). This situation poses a dilemma for teachers who wish to incorporate mathematical play into their classrooms, with thin empirical basis for its value. Eventually, we hope our research will influence classroom instruction, but our first step is to examine how meaningful mathematics is joyfully engaged in school-age children's construction play.

Prior work: Construction play and children's mathematical learning

Prior research on how mathematics is engaged through play highlights construction play as mathematically generative (Sarama & Clements, 2009; Seo & Ginsburg, 2004). In construction play, children design with objects and engage in higher-level thinking as they solve problems that emerge from the constraints of construction materials (Bergen, 2009; Forman, 2006). There is evidence that preschool children who engage in construction play might do better in math later on (Stannard, Wolfgang, Jones, & Phelps, 2001). Additionally, older children who do well on construction tasks also tend to perform well in mathematics (Casey, Pezaris, & Bassi, 2012). While most of these studies were conducted in laboratories without intentionally designed materials, pedagogical theorists such as Maria Montessori have made much work of these ideas. In fact, Montessori schools in the U.S. leverage exploration in construction play as a primary aspect of their preschool curriculum. These pedagogies specifically leverage design of materials to promote scripted mathematical exploration.

However, we break from Montessori both in the age of children studied, and in what we count as play. First, the mathematical sensemaking of young children likely looks very different than the mathematical sensemaking of older children (Clements & Battista, 1992). Second, the nature of scaffolding in Montessori—where teachers model how to engage with materials (Lillard, 2013)—runs counter to our notions of play. Our conceptualizations of play feature children's agency in exploration, self-selection of goals, and self-direction in how to accomplish them (Wing, 1995). Just as mathematics education research highlights how agency benefits learning, we hypothesize that children's agency in play will lead to rich exploration of construction play materials. We argue that examining children's self-directed construction activity with mathematically structured objects is likely to provide rich examples of children's emerging mathematical sensemaking.

Surprisingly, there are few studies of upper-elementary and middle school aged children's construction play with materials explicitly designed to direct children's attention to mathematical concepts — perhaps because foundational scholars like Piaget (1946/1962) and Vygotsky (1978) focus on sociodramatic play as the leading activity of this age group. This has created a missed opportunity, as engagement with sophisticated materials can lead to productive mathematical exploration for older children (Papert, 1980). In fact, the mathematical generativity of construction play emerges from the material constraints that lead children to experience trouble as they work to achieve their goals (Bergen, 2009; Papert, 1980). Our research thus asks: *How do school-age children engage in mathematically generative play in a designed out-of-school mathematical playground?*

Conceptual framework: Mathematically generative play as problem-solving through trouble and repair

Theoretical framework

To understand how upper elementary and middle school children engage in mathematically generative play, we follow Vygotsky's (1978) theory of *mediated action*. This theory holds that all knowing and doing is mediated by cultural tools, including language and physical objects. The objects at MOAS were intentionally designed to facilitate encounters with mathematical concepts (Wertsch, 1998). When the *what if* aspect of play intersects with the mathematical practice of conjecturing, we posit that children are working just beyond what they already know and understand. Questions like, *what if I move this tile here? what if I rotate it this way?* support explorations and encounters with mathematical concepts. When this happens, we see this as evidence of children operating in their zones of proximal development (ZPDs; Vygotksy, 1978), making these moments of potential learning. Of particular interest are moments where children (a) encounter trouble in meeting their self-generated construction play goals and then (b) make multiple attempts to repair that trouble, whether independently as the materials themselves act as scaffolds or with a more expert other who poses questions or models solutions. We consider these episodes of trouble-and-repair to be examples of problem-solving, as children's work in their ZPD inherently involves problematic experiences for children relative their current understandings.

Mathematical learning in play

Foundational to this study is a view of *math embedded in play* (Ginsburg, 2006). Notably, Seo and Ginsburg (2004) looked at young children (<5 years) playing with mathematically rich objects and inductively developed explanatory categories for their activity. In doing so, they identified mathematical ideas children could notice, even when children did not frame their activity as math.

To understand children's sensemaking in play, we attend to children's problem-solving during goal-based play. This is both a theoretical and a methodological choice. Theoretically, problem-solving requires an end goal, so identifiable goals help us analyze it. Methodologically, we count construction play to begin when children set goals (Hutt, 1979; Vandenberg, 1980), and these are often most visible when they are not met. We acknowledge that (a) children engage in exploratory play without construction goals, (b) children have goals we cannot identify, and (c) children may achieve their goals without experiencing trouble and may be engaging important mathematical ideas. Pragmatically, we analyze the cases where the problem solving is most visible and omit these cases where it is less so. As a result, we focus only on episodes in which children set visible goals, experience trouble, and make multiple attempts at repair, as these are low-inference cases for the phenomenon of interest.

Research design: Understanding problem solving and mathematical generativity in play

To explain our logic of inquiry, we describe our data collection procedures and identify our units of analysis. We then outline our data sampling and analysis methods for our research question, *how do school-age children engage in mathematically generative play in an out-of-school mathematical playground?*

Site selection

Research took place at a mathematical playground called Math On-A-Stick (MOAS) at the Minnesota State Fair. Data were collected over the full ten-days of the fair in 2016. MOAS was open to any fairgoer. It was a pleasant, shady space in a relatively quiet corner, containing nine tables, each of which had unique mathematically structured objects that children could use as they pleased along with volunteers who could facilitate play. These objects included pattern machines, various tiling pentagons, tessellating turtles, and 6x5 egg crates with colorful plastic eggs. Children's voluntary participation, their freedom in how (and how long) to engage with the exhibits, and the mathematical richness of their design support a study of children's sensemaking as they encounter mathematical ideas in unexpected ways.

Data collection

To understand the mathematical generativity of children's play, data collection aimed to capture children's perspectives of their activity in MOAS exhibits. We recruited 348 children to participate. They answered brief intake surveys and then went about the playground with Go-Pro video recorders mounted on their heads, aimed downwards and slightly forward to capture the children's and adults' talk, gestures, and object manipulation. This view captures the locus of the children's attention when they are playing with objects: When the child looks up, the camera shows what they are looking at. Even slight glances at other children playing are captured. This video record supports inferences about children's attention and interest. As participants exited MOAS, we conducted brief interviews about their experiences. The video data serve as the primary data for the study, with the survey and interview data used primarily to select participants based on different reported characteristics and experiences.

Data corpus

The 348 participants ranged in age from 4 to 16 years old. The average MOAS visit lasted 26 minutes (sd = 0.007), with median visit per exhibit at approximately four minutes. Out of the 348 participants, this study focuses on upper elementary and lower secondary aged children's play (7 – 12 years old, n = 277, visit length: m = 28 minutes , sd = 0.01). This limits the variation in children's development and addresses a gap in the literature, as this age group's out-of-school mathematical learning is understudied.

Case selection

To illustrate how our methodology supports an analysis of mathematically generative play, we present the case of one child at the egg exhibit at MOAS. Olivia (a pseudonym) was an eight year old girl who played at the egg exhibit for over 12 minutes, much longer than the average stay time. We conjectured that sustained engagement at one exhibit might support increasingly complex design goals. We focus on Olivia's last construction at the egg table, where she had a goal of making a heart design using pink plastic eggs in a 6x5 egg crate. This episode of goal-based activity lasted approximately seven minutes, meaning that this single design took her longer than most children remained at this exhibit. Because Olivia's mother was attentive while she engaged in construction play, their interaction was analytically useful, because it gave us access to Olivia's thinking.

Unit of analysis: Episodes of goal-based activity

Because we seek to identify mathematically generative play, we distinguish between (a) play that produces mathematical ideas that the children *might* have noticed and (b) play that produces mathematical ideas that we can empirically argue that they noticed. In other words, while we can certainly infer mathematical ideas from children's play activities, we seek to identify the mathematics that *children attend to in play*, even if it is not formally named.

As a preliminary data reduction strategy, we focused on *episodes of goal based activity,* parsing our subset of videos into these episodes. To allow enough time for exploratory play and to seek examples of sustained engagement, we sampled video from children who stayed at an exhibit at least five consecutive minutes. Out of the 277 school-aged children in our sample visiting the many exhibits at MOAS, this data reduction yielded 395 episodes of exhibit level activity (~50 hours of video data). We reasoned that longer episodes were more likely involve multiple attempts at reaching the goal. To determine the start of goal-based activity, we attended to both *explicit goals* (e.g., the statement "I'm going to make a dog!") and *implicit goals* (e.g., making a "blank slate" by ruining what was previously made). To determine the end of goal-based activity, we attended to when children: (a) *discard* (e.g., child destroys what made); (b) *abandon* (e.g., walks away without finishing); (c) *preserve* (e.g., having parents take photographs), or (d) *share* (e.g., soliciting adult praise). If, after preserving or sharing, the child continued to work on the same object to refine (but not dramatically alter) their work, we considered this part of the same goal-based activity, reapplying the criteria for the end of goal-based activity to determine when the activity truly ended *for the child*.

Second, we further reduced our data by identifying *episodes of trouble-and-repair.* Because we are interested in the mathematical learning supported by play, these moments made visible times when children had trouble reaching a goal, along with their strategies and resources for repairing the trouble. Trouble was located through children's: (a) repeated attempts; (b) use of multiple strategies; and (c) expressions of frustration or confusion. While some exhibits afforded longer play than others, analyses show that two minutes of goal-based activity often captures multiple attempts at repairing a problem. We selected Olivia's case by examining the duration of episodes of trouble in our data set, with hers being and outlier in sustained engagement in repair work.

Analytic methods

To examine how school age children—and Olivia in particular—engaged in mathematically generative play, we use methods of interaction analysis (Jordan & Henderson, 1995) to identify when children experienced trouble and worked to repair it. *Trouble* occurred when participants' expectations were broken (p. 69), such as when children's patterns and designs did not emerge as they intended. This approach allowed us to attend to both verbal and nonverbal features of interaction. Indicators of trouble include when children explicitly asked for help (e.g., "How do you make a circle?"), asked for formative feedback (e.g., "Does this look like a heart?"), expressed frustration (e.g., "Ughhhh"), gestured inquisitively, or made trial-and-error revisions (e.g., moving pieces to make a pattern "look right"). When children persisted and made multiple repair attempts using the mathematical features of the objects, we view this as *mathematically generative play*, even if the trouble never was fully repaired.

Findings

Children engage in complex problem-solving during play at MOAS, and this often takes the shape of repair work. To illustrate how repair work supports children's grappling with mathematical concepts and approximates disciplinary practices, we share a brief episode in which trouble occasioned mathematically generative construction play. Olivia worked with her mother for seven minutes to achieve Olivia's play goal of *making a heart* in a 6x5 egg crate. Over the course of their activity within the play goal, Olivia articulated aspects of the heart that were important to her, which manifested as emergent problems in play. She refined her account of the emergent problem three times, with each articulation of the problem indicating an element of Olivia's mathematics sensemaking: (1) "Does this look like a heart?" (notices shape without midline symmetry), (2) "Ohhh, it's crooked" (focuses on asymmetry), (3) "Where's the middle?" (focuses on egg crate as a 6x5 grid with a midline). The remainder of this section analyzes how Olivia's question of "Where's the middle?" launches Olivia into her ZPD as she persists in her play goal. Due to the density of gesture, which is pivotal for understanding sensemaking in constructive play, the transcript below explores 30 seconds of interaction from this last phase, beginning when she refined her problem to "Where's the middle?" and then began to structure a solution to the problem. Given limited space, we use only one still image from the recording for each turn, though the multimodal analysis we conducted is considerably more complex. The extra punctuation denotes speech characteristics like breaks -, elo::::ngation, ((*notes about gesture*)), volume, and *intensity.*

Transcript Excerpt

	Talk and ((activity))	Screenshot	Analytic Interpretation
(38)	**Olivia:** Whe::re's the- ((*moves egg to slots on either side of the actual middle in the egg crate*, then *rests egg on actual middle*)) middle?		[00:28:59.20] Olivia and Mom are in a facing-formation with joint attention on the egg crate. Olivia asks for assistance to find the practical middle in an egg crate that offers a 6x5 grid. By moving the egg, Olivia draws Mom's attention to the absence of a midline in her current use of the crate.
(39)	**Mom**: Well that's the problem, there's- there's really no middle because this is the middle ((*finger runs along middle line her daughter had just touched then turns away to talk with another adult*))		[00:28:59.28] Mom scaffolds attention to missing mid-line symmetry for the entire carton, then turns away, breaking the facing formation and ending joint attention to the carton as a grid.
(40)	**Olivia**: ((*moves body to align with side having five slots, grabs crate, then rotates body back and held crate to her original position*))		[00:29:08.16] Olivia uses her whole body to investigate the practical middle of the other side of the crate.
(41)	**Olivia**: Oh maybe if we- *Mommy::* ((*grabs Mom's arm to get her attention, then rotates crate back to original position*))		[00:29:10.14] Olivia bids urgently for Mom's attention, pulls her body back, and restores their former facing-formation.
(42)	**Mom:** ((*turns back towards Olivia*))		Mom leaves adult conversation and turns her body to restore former facing-formation with her daughter.
(43)	**Olivia**: Maybe if we *tu:::rn* it ((*rotates crate again so Mom can see the contrast*))		[00:29:13.14] Olivia demonstrates her discovery to Mom by re-enacting the rotation of the crate while her mother is in their facing formation

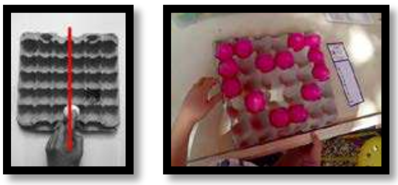
(44) **Mom**: Oh <u>yeah</u>, good idea. Good thinking honey!

Mom <u>recognizes Olivia's solution to the problem of a missing mid-line</u> (Turn 39) and praises her ingenuity with a term of endearment.

(45) **Olivia**: *Oh::::*, no::w I can do:: it! ((*takes all of the eggs out of the carton*))

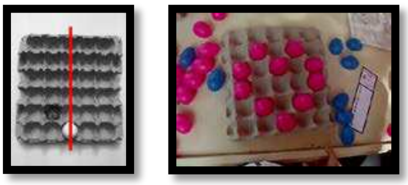
[00:29:27.24] Olivia clears the crate/grid and starts again to make a symmetric heart with a point/egg that falls on a true midline. Rotating the crate makes this solution possible.

Over the course of this episode of goal-based construction play, Olivia structured a new problem with new affordances for symmetry through her visible attention to aligning *actual middles* (i.e., midline symmetry is at the middle of six, Figure 1) with *practical middles* (i.e., midline symmetry is at the middle of five, Figure 2).



(a) (b)

<u>Figure 1</u>. No practical midline symmetry (a) with heart centered on practical middle (b).



(a) (b)

<u>Figure 2</u>. Practical midline symmetry (a) with heart centered on practical middle (b).

When asking "Where's the middle?" Olivia placed her egg on the crate's midline of symmetry (Line 38, Figure 1a), although it was not a *practical* middle because she had the crate rotated such that the side with six slots was parallel to her body and the egg could not rest in the center. After asking this question, Mom scaffolded Olivia's understanding saying, "There is no middle," while tracing the mid-line of symmetry with her hand (Line 39, Figure 1a). Importantly, Mom did not tell Olivia how to resolve the problem of *no middle* but rather temporarily disengaged by breaking the facing-formation (Line 39). Olivia then moved her body around the crate, making herself to parallel to the side with five egg slots. She then jointly rotated both herself and the crate back to her original position (Line 40), resulting in the crate being oriented so that its mid-line of symmetry and the practical middle for making a heart *with a point* became aligned (Figure 2a). Olivia then made a bid for Mom's attention (Line 41) and, re-establishing the facing formation (Line 42), Olivia re-rotated the crate so that Mom could see the change (Line 43). Here, both Olivia and Mom demonstrated (Lines 44 and 45) that they now knew they had re-structured the problem space to align the practical and actual middles of the egg crate grid. Once Olivia removed the eggs from the crate grid, she exclaimed, "Ohhhhh, now I can do it!" Olivia then created her heart-with-a-point unproblematically (Figure 2b).

This episode gives a glimpse into the child's perspective of trouble experienced in play and shows the potential for mathematical generativity in goal-based activity. First, Olivia iteratively refined her definition of the problem from *not looking like a heart* to eventually a problem that suggested a solution path: find or make a middle. Second, Olivia used important disciplinary practices (National Governors Association, 2010), such as (a) *attending to precision* as she refined her goal to be a heart *with a point,* (b) *looking for and making use of structure* as she transformed the problem space to have *matching practical and actual middles*, and (c) *using appropriate tools strategically* as she began to *treat the egg crate as a grid*. Third, as Olivia played, she attended to mathematical properties of the egg crate — the evenness and oddness of the two sides, the presence or absence of a true middle — and relations between them. We posit that this linking of mathematical properties facilitates deeper understanding of symmetry. Although it is not the focus of this analysis, we note that Mom effectively scaffolded Olivia, taking up her questions while also allowing her to maintain control of the decision-making process and deciding when the heart was "Done, Mommy!" (Line 63, not shown).

Conclusion and implications

This study uncovers the potential for mathematically generative play, finding that construction play with mathematically structured objects can facilitate children's attention to mathematical properties of objects as they overcome trouble to meet practical goals. This study underscores the generativity of seemingly simple ideas (such as middles and symmetry). Future research should continue to explore this generativity, as well as examine how adults can better scaffold children's sensemaking of key mathematical ideas. Our aim for this research program is to develop empirically-informed design principles that bring joyful disciplinary engagement into classrooms through investigating how school-age children's play can be mathematically generative. By building an empirical basis for disciplinary enjoyment and engagement through play, this work will enable a broadening of mathematical participation in two ways. First, the mathematical ideas that children find challenging in contexts other than paper-and-pencil activities might surprise the adults who guide their thinking (Ginsburg, 2006). In other words, mathematics does not seamlessly transfer between scales and modalities (Hall, Ma, & Nemirovsky, 2015), so intentional scaffolds must be designed as we create wider and deeper ecosystems for mathematics education. Second, children's competent informal mathematical thinking is often not recognized by adults (Ginsburg, 2006). By capturing examples of children's productive, informal mathematical thinking, this research can facilitate mathematics educators' recognition of swaths of competence. By underscoring the inherent challenge of seemingly simple ideas such as middles and symmetry, adults can be better attuned to helping children make meaning of key mathematical ideas.

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