International Society of
the Learning Sciences

Using Reference Models as a Springboard for Ontological Innovation: Analyzing a Central Theory Building Move in the Learning Sciences

Hillary Swanson, Utah State University, hillary.swanson@usu.edu,
Mariana Levin, Western Michigan University, mariana.levin@wmich.edu

Abstract: The development of learning theory is of core importance to the learning sciences. Despite its importance, little is understood about our own theory building processes. This paper analyzes one case of theory development to better understand a particular theory building move associated with design-based research: ontological innovation. The case features the invention of a new category of knowledge and therefore exemplifies ontological innovation. Conducting a retrospective analysis, we identify seven steps, which were critical to the development of the new category of knowledge (meta-theoretic competence) in response to an existing reference model (meta-representational competence). These steps suggest the coupled nature of data and theory, and the importance of the researcher's own knowledge, which they bring to bear on their theory building practice.

Introduction

The development of learning theory is of core importance to the learning sciences (diSessa, 1991; Roschelle, 2021; Sandoval, 2014). In the last thirty years there have been many exciting developments in the field that have fundamentally shaped how we view the very nature of knowledge and learning processes, including a turn towards recognizing how learning is embodied in nature and socially and culturally organized (Abrahamson et al., 2020; Nasir et al, 2021; Nathan, 2021). Many of our most impactful advances have come via the strong commitment to interdisciplinarity in the learning sciences: building upon advances in cultural studies, sociology, anthropology, and educational neuroscience, to name a few. Together with these perspectives, the field has developed ethnographic and design-based methodologies that provide sources of insight into real-time learning processes (Bakker, 2018; Brown, 1992; Cobb, et al, 2003; diSessa, Sherin & Levin, 2016; Hall & Stevens, 2016). One of the hallmarks of theory building in the learning sciences is that theories that are developed in the context of real-world learning and performance data can generate novel theoretical ideas of wide scope. diSessa and Cobb named these novel theoretical ideas *ontological innovations* — "theoretical constructs that empower us to see order, pattern, and regularity in the complex settings in which we conduct design experiments" (diSessa & Cobb, 2004, p. 84). Ontological innovation can also be viewed as a theoretical move involving the recognition of new theoretically-relevant categories based on observations in data.

While it is widely agreed that the development of learning theory is an important goal, processes of theory development in the learning sciences are still not well understood. By its nature, theory building in the learning sciences has a strong commitment to being accountable to the moment by moment details of learning processes. Given this commitment to theorizing involving emergent categories based on empirical data, many moment-by-moment analyses of learning interactions describe the process of building theory as being informed by Grounded Theory (Glaser & Strauss, 2017/1967). However, while Grounded Theory shares with learning sciences researchers the commitment to extracting categories for research from empirical data, the description of the theory-building process is not well-tuned to modeling moment-by-moment learning processes (Parnafes & diSessa, 2013). Previous descriptions of theory building within the learning sciences have resulted in descriptions of general processes, such as researchers engaging in *observe*, *schematize* & *systematize* cycles (diSessa, in Parnafes et al., 2008; diSessa & Levin, 2021) in the case of generating new theoretical constructs, and general phases of theory building, such as *incubation*, *theory negotiation*, and *theory appropriation* phases (described in Parnafes & diSessa, 2013) in the case of analyses that extend and elaborate an existing theoretical construct. While these case observations offer helpful heuristics, the details of how these cycles and phases function require more elaboration, and exemplification in a broader set of instances. Our aim in this paper is thus to offer a case analysis of a theory building process, focusing in particular on the way existing theoretical constructs can serve as reference models that then lead to ontological innovations. The case we present comes from an analysis of the theory building process of the first author.

Theoretical foundations

We begin by describing the theoretical framework that guided the theory building work featured in the case study: knowledge in pieces (diSessa, 1993). We then describe the theoretical framework that we use to characterize the theory building process: epistemic forms and games (Collins & Ferguson, 1993).

Knowledge in pieces

Knowledge in pieces (KiP) is a name for a broad class of theoretical models of knowledge—models in which knowledge is seen as consisting of a *complex system of elements of diverse types*. This broad epistemological perspective grew out of studies of individuals reasoning about the physical world. DiSessa showed that the way individuals thought about physical phenomena was well-modeled by a large number of knowledge elements that were often inarticulate and applied in a manner that depended sensitively on the context at hand (diSessa, 1993). This formulation of the organization of knowledge has strong implications for instruction: instead of replacing "misconceptions" about how the world works, instruction in formal physics needed to engage individuals' intuitive knowledge and re-contextualize it more appropriately for given contexts.

Knowledge in pieces has influenced studies in many other domains beyond intuitive physics, including algebra (Izsák, 2000; Levin, 2018), multiplication (Izsák, 2005), proportional reasoning (Izsák, Beckmann & Stark, 2021), calculus and linear algebra (Adiredja, 2014; Adiredja & Zandieh, 2020), geology (Barth-Cohen & Braden, 2021), and computer science (Lewis, 2012). As well, the framework has influenced the study of naive epistemology (Hammer & Elby, 2002), the role of emotions in cognition (Gupta, Elby, & Danielak, 2018), and ideologies about race and racism (Philip, 2011). Common themes across this body of work are that (a) intuitive knowledge based on experience in a domain provides a rich source of resources for learning, (b) the recognition and use of such knowledge resources are not necessarily articulate and that (c) the activation and use of knowledge resources depends sensitively on context. More extensive lists of principles and counter-principles for studying knowledge in a way that is consistent with knowledge in pieces is given in diSessa, Sherin & Levin (2016).

Ontological innovations may occur at the system level or at the element level, and sometimes both. A number of novel knowledge categories have been proposed over the years, including *phenomenological primitives* (p-prims; diSessa, 1993), *explanatory primitives* (e-prims; Kapon & diSessa, 2012), *naturalized axioms* (Philip, 2011), and *symbolic forms* (Sherin, 2001). More extensive knowledge systems such as *coordination classes* (diSessa & Sherin, 1998; diSessa & Wagner, 2005, Levin & diSessa, 2016) have been generated to explain how individuals obtain measurable information from the world and *strategy systems* (diSessa & Levin, 2021; Levin, 2018) have been used to explain how individuals coordinate knowledge in problem solving processes. In this paper, we show how an existing knowledge category (meta-representational competence; diSessa, 2004) is used as a springboard for ontological innovation.

Epistemic forms and games

Collins and Ferguson (1993) characterized the theory building work of scientists using a framework they called epistemic forms and games. Epistemic forms are abstract structures or templates, which scientists fill out in answering a particular research question. A form guides the game that is played to fill it out, in the same way that the cross-hatch structure of tic-tac-toe imposes certain constraints on how the game can be played. The completed form embodies theoretical knowledge, the form and game are therefore connected with knowledge and knowledge building. Collins and Ferguson present three kinds of epistemic games: structural analyses, functional analyses, and process analyses. They describe examples of typical forms produced by games within each category (e.g., lists, stage models, system-dynamics models), and unpack some of the moves in the associated games. For example, a scientist might play the list-building game to produce a list when their research question is something like "what is the nature of x?" and x can be decomposed into a smaller set of elements (e.g., a list of material substances found in the world). If the items on their list can be characterized in terms of multiple dimensions, the scientist might transfer to a cross-product or table game (e.g., the periodic table of the elements). In our analysis, we use the framework of epistemic forms and games to schematize the moves made by the first author in her theory building trajectory.

Methods

In order to understand the smaller steps involved in ontological innovation, we conducted retrospective analyses of our own theory building processes as case studies. In this paper, we report on the findings from the analysis of one case study. The case study was conducted by the first author; the Analytic Process and Findings sections are therefore written in first person. The case study examined the development of a knowledge category called meta-theoretic competence (MTC), which was developed to characterize 8th grade students' productive engagement in theory building, in the context of a science elective course (Swanson, 2022).

Context of research on student theory building

Data were collected in the context of a class designed for 8th grade students – the "Patterns" class – that met three mornings a week for forty minutes, both fall and spring semesters. The Patterns class was designed to engage students in the construction and refinement of a unique kind of theoretical artifact: a pattern theory. Pattern theories were essentially written descriptions of patterns in behavior underlying phenomena across domains. For example, a pattern of threshold can be seen in both the tipping point of a tower of blocks and the testing of a person's patience. Pattern theories were meant to serve as conceptual footholds for moving towards more mathematical, aggregate-level understandings of phenomena through the lens of dynamical systems theory. Twenty-one students participated in the course (11 girls, 10 boys). The teacher was also the researcher. Data were collected in the form of video footage and student work.

The course included units on patterns of threshold, equilibration, exponential growth, and oscillation. Each unit followed the same general sequence, meant to embody a simplified process of theory building and refinement. Students began by exploring two phenomena that exemplified the focal pattern. They were then invited to individually write descriptions of the pattern in behavior the two examples had in common (these were their first theory drafts). They evaluated their pattern theories against a third example and then revised their pattern theories to either clearly include or exclude the example. They then generated lists of examples that followed the pattern and discussed those examples in the context of a whole class debate. They were then invited to revise their thinking and write third and final pattern theory drafts.

For example, in the threshold unit, the students explored phenomena that exemplified the pattern of threshold (adding coins to a spaghetti bridge until it broke; adding drops of water to the surface of a coin until it overflowed) and then wrote descriptions of the pattern in behavior demonstrated by both examples. They then compared their pattern theory with another example (adding scoops of salt to a cup of water until a submerged egg floated) and refined their theory to more clearly include or exclude the example. They generated a list of additional examples that followed the pattern (e.g., "bothering someone till they burst," "getting a haircut") and argued about the fit of those examples with their classmates in the context of a whole class debate. By the end of the unit, they had refined their pattern theories (e.g., "adding something to something until it changes").

Analytic process for the case study of researcher theory building

The aim of the research described above was to characterize students' theory building in the context of the Patterns class. My KiP orientation made me interested in characterizing their theory building at a microanalytic scale and in understanding how their engagement in the activities I'd designed could be seen as seeds of scientific theory building. This orientation also placed the goal of modeling naive knowledge in my awareness, though it was not initially foregrounded in my thinking.

In order to understand my own process of theory development, I conducted a retrospective analysis of several sources of data. The first was a document containing reflections, which I had written throughout the course of my analysis of student work and video transcript. The second source consisted of the notes I had written on student work and video transcripts of class activities. I used the reflection document to reconstruct a narrative of my process at a level of detail that captured milestones in the development of my theoretical artifact (e.g., initial characterizations of the data, identifying a reference model). I used my notes on student work and transcripts to illustrate my narrative with specific examples. I read through the narrative of my process and partitioned it into seven steps, to capture what appeared to be key moves in the development of my theory. I present the narrative of my theory building process below, unpacking each of the seven steps in chronological order.

Findings

In this section, I describe the theory building process that emerged as I worked with the data. I partition my process into seven steps. The first step maps with Parnafes and diSessa's *incubation* stage. The second step maps with their *theory negotiation* stage. The third through seventh steps align roughly with their *theory appropriation* stage and offer a closer look at how a reference model can act as a springboard to ontological innovation.

Step 1: Examining data and assigning initial characterizations

The initial goal of my inquiry was broad: to understand the nature of student engagement in theory building in the Patterns class. I began by looking through the data and characterizing, at no particular grain size, elements of student work and activity, which seemed relevant to elements of theoretical science. In this way, my work during this step aligns with both Parnafes and diSessa's *incubation* stage, and diSessa's *observe* and *schematize* stages. As in the *incubation* stage, I began by looking across the data without hunting for evidence of existing theoretical constructs. However, in contrast with the *incubation* stage, my initial look wasn't purely open. Instead, my initial interaction with the data resembled a combination of the *observe* and *schematize* stages. I viewed my data with the expectation that students would be doing productive things related to theory building, and I loosely schematized the productive things I saw the students doing.

For example, to students' pattern theories, I added notes such as "general language" and "pattern in behavior." To the margins of the video transcript, I added notes such as "offered intuitive causal explanation" or "challenged explanation on grounds of logic." In reviewing my notes, it became clear that many of my characterizations of student theory building could be categorized as *theory building moves* and those that were not (e.g., "general language") could be recast as moves (e.g., "used general language"). I conceptualized these theory building moves as *resources* for theory building. Though it seemed advantageous to characterize the resources as moves where possible, I felt using the term "resource" had an advantage over "move," in that it was less constrained and could involve a wider range of seeds for theory building, such as general aesthetics and intuitive explanations, which I also saw in the data. This move in my own theory building process is related to diSessa's *systematize* stage. My shift in schematization of the data inspired a clarification of my research aim and my question became more focused: "What resources do the students bring to their productive engagement in theory building in the Patterns class?"

Step 2: Identifying a reference model

I recognized that answering this question would yield a collection of resources students had for the construction of pattern theories. This cued to memory a collection of resources for engaging in another set of scientific practices, which already existed as a category of knowledge in the KiP framework. This category of knowledge was meta-representational competence (MRC; diSessa, 2004). MRC supports work with representations, which transcends their mere production and use. It includes skills for the invention and design of new representations, for their critique, for understanding the purposes of representations (both in general and in specific contexts), for explaining representations, and for learning new representations quickly with little intervention. MRC includes resources that are "gradually developed through cultural practices in and out of school." It is therefore concerned with naive knowledge that extends beyond the realm of intuition.

DiSessa and colleagues documented resources for the invention and design of new representations, such as the ability to draw and sensitivity to representational possibilities of visual elements (e.g., color, length, width). Resources for the critique of representations include criteria such as completeness, precision, and parsimony. It seemed sensible to use MRC as a reference model for organizing a class of resources for scientific theory building. Both models are concerned with a broad range of resources on which students can draw to engage in scientific practices, and both are concerned with elements of naive knowledge, which may have their origins in social interaction and schooling. To make the connection between my work and this reference model explicit, I decided to call this new category of knowledge "meta-theoretic competence."

Step 3: Abstracting the epistemic form underlying the reference model

Meta-theoretic competence (MTC) is a new category of knowledge. It is therefore an ontological innovation. To develop MTC, I drew inspiration from an existing ontology: MRC. In order to understand how my ontological innovation was seeded by a reference model, it is helpful to view my theory building activity through the lens of epistemic forms and games. My theory building was an epistemic game, the moves of which were driven by the epistemic form I was trying to produce. My target form was inspired by MRC, but MRC itself was not my target form. My target form was the *general structure underlying MRC*, which is a list of resources for productive engagement in a particular activity. Whereas MRC is concerned with students' engagement with representations, my concern was theory building.

The form I sought to produce was therefore a list of resources for productive engagement in different aspects of theory building. It is worth calling attention to the fact that I sought the general form underlying MRC and used that as the template to guide my theory building work. This move is likely idiosyncratic and reflective of my own interest in the nature of scientific theory building and my attention to schematizations of theory building that have previously been described in the literature, including epistemic forms and games.

Step 4: Organizing initial characterizations according to the epistemic form

Having determined the epistemic form I would need to fill out to answer my research question, I turned to organizing the initial resources I had identified in my data in the form of a list. Because MRC partitions resources according to aspects of work with representations such as invention, critique, etc., I anticipated that I would ultimately partition the elements on my list according to parallel aspects of theory building. However, at this point in time, I didn't know what those categories should be, so I merely wrote the list in an order that matched my notes in the data.

Step 5: Refining the emerging model by creating categories

Looking across the resources, I thought of categories of theory building activities that could be used to organize my list. The categories that initially emerged were: theory articulation, evaluation, and refinement, and specification of a theory's domain of application. This move also connects with diSessa's *systematize* stage. To see how this worked, I organized the elements on the list according to these four activities.

Step 6: Going back to the data and reanalyzing it in terms of the model

With my revised template, I went back to the data and in addition to checking the appropriateness of my existing list of resources, I looked carefully for other examples in the data that might be resources for articulating, evaluating, or refining theories, or for specifying a theory's domain of application. I added resources to the list, filling out the template more fully. The activity of this step corresponds again with diSessa's *observe* and *schematize* stages.

At this point, I created two copies of the list. The first copy presented the resources simply in their categories as a partitioned list. For the second copy, I expanded the list by describing each resource in detail and illustrating it with student work or segments of transcript from the Patterns class.

Step 7: Further refining the model by changing the epistemic form

Working more closely with the data and explaining how it illustrated each resource on the list surfaced a pattern I hadn't previously noticed. What the examples revealed was that there was another dimension along which the theory building resources could be organized. This was a dimension of characteristics that were particular to Pattern theories. These dimensions included deeper structure, generality, and causal relationships. I tried reorganizing the list of theory building resources according to these aspects. The result was unsatisfying because, while it highlighted which aspect of pattern theories a resource could be used for, it obscured which theory building activity it corresponded with. As I rearranged the examples from my illustrated list, it became clear that the resources I had found were accountable to both dimensions (aspects of pattern theories and theory building activity). For example, in some of the data, students were demonstrating resources for *articulating causal relationships*, while in other examples, they were demonstrating resources for *evaluating a theory's level of generality*.

This led me to wonder if a cross-product form might be used to organize the theory building resources. I constructed a table to see what it would look like to have theory building activities in rows and pattern theory characteristics in columns. In my data, I found examples of resources for the articulation, evaluation, and refinement of pattern theories along the dimensions of deeper structure, causal relationships, and generality. It didn't make sense to cross the specification of the domain of application (essentially naming examples of things that followed the pattern) with characteristics of pattern theories. The product of this with deeper structure, for example, didn't mean anything. I struggled with what to do with "specifying the domain of application." It was a key component of the pattern theory building process and resources for it would need to be included in my ultimate model of MTC demonstrated by students in the Patterns class.

Because "specifying the domain of application" was so particular to pattern theory building, I decided to try moving it from the dimension of "theory building activity" to the dimension of "aspects of pattern theories." This shift required a refinement in the names of the two dimensions, as the table now consisted of a dimension of stages of theory building (articulating, evaluating, and refining - in rows), crossed with pattern-theory-specific elements (deeper structure, generality, domain of application, and causal relationships - in columns). A reduced version of the table is shown below (Table 1), to give the reader a sense of the kinds of resources that were included in the category, and to illustrate how they were organized in tabular format.

Table 1. Resources for theory building, which belong to the category of meta-theoretic competence

Stages/Pattern Characteristics	Deeper Structure	Generality	Domain of Application	Causal Relationships
Articulating	Articulating the pattern in behavior	Using general language to describe a pattern in behavior	Generating examples from physical and psychosocial domains	Articulating intuitions about

	instantiated by examples			causal relationships underlying behavior
Evaluating	Comparing examples based on similarities and differences	Questioning the generality of the language used	Challenging and defending the fit between examples and pattern theories	Challenging and defending the logic of intuitions
Refining	Removing surface features from the pattern theory	Removing context-specific features to make the pattern theory applicable to more examples	*There was no opportunity in the course for students to demonstrate resources for this category	Responding to challenges by unpacking or modifying ideas

In the language of epistemic forms and games, my shift from a list form to a table form is an example of a "transfer" from one epistemic form to another. This is an appropriate move in playing an epistemic game, if it becomes apparent that the original form is insufficient. Because it is focused on the larger organization of the elements, the activity of this step corresponds with diSessa's *systematize* stage.

High-level sketch of the theory building process

The theory building process described above consisted of seven steps: 1) examining the data and assigning initial characterizations, 2) identifying a reference model, 3) abstracting the epistemic form underlying the reference model, 4) organizing initial characterizations according to the epistemic form, 5) refining the emerging model by creating categories, 6) going back to the data and reanalyzing it in terms of the model, and 7) further refining the model by transferring to a new epistemic form. Figure 1, below, illustrates the seven steps of the researcher's theory-building trajectory.

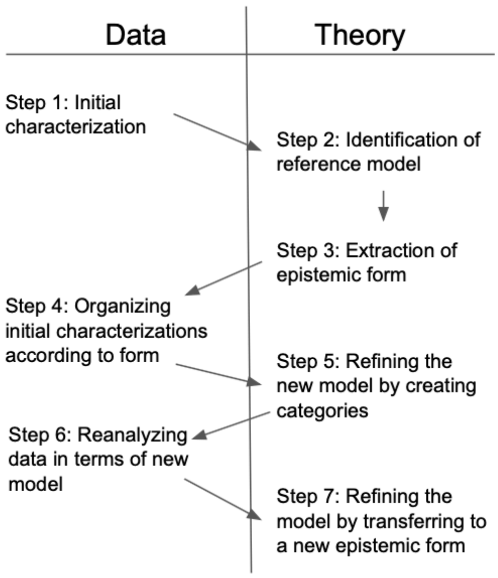

Figure 1. High-level sketch of the researcher's theory building trajectory.

Looking across these seven steps gives us a general sense for how the reference model acted as a springboard for ontological innovation in the examined case. There was a general pattern in which the researcher moved back and forth between data and theory, but the two were never quite separate. When the researcher gave the data an initial look, for example, she was looking at the data through the lens of her theoretical orientation with the expectation that the students would be doing productive things with respect to theory building. She therefore saw characteristics of artifacts and activities that might be productive resources for theory building and lifted those out of the data by schematizing them as moves, for example. When she looked across the list of resources she had identified, she cued to memory a potential reference model, based on perceived similarities in structure and function. She drew this model from a network of familiar models. Because she had read the work of Collins and Ferguson and viewed her own scientific inquiry through the lens of epistemic forms and games, she was motivated to extract the epistemic form of a list from the reference model. She then used the form as a template for organizing her initial characterizations (or in the words of diSessa, she used the template to systematize her initial schematizations). Because she valued the construction of theory grounded in data, she continuously tested her model against the data and refined it so that there was a close fit.

The sequence illuminates a general pattern of moving back and forth between the data and the form, where the researcher keeps the one in mind, while looking at the other. It also shines a light on the importance of the knowledge the researcher brought to her theory building, or the role of her own meta-theoretic competence. Her theory-building knowledge included her epistemological orientation, which, typical of a KiP perspective, viewed naive knowledge as potentially productive and likely containing seeds for scientific theory building. Her theory-building knowledge also included a network of existing KiP models, which provided a number of possible reference models. Her theory-building knowledge also included an epistemological stance on the nature of scientific theory building, which was that scientists played epistemic games to fill out epistemic forms, which were general structures underlying particular models. She therefore thought to draw the deeper form out of the reference model and use it as a template for driving her epistemic game. Finally, her theory-building knowledge included an aesthetic for building theory grounded in data. All of these elements of her theory-building knowledge came together in her attempt to make more formal sense of her students' engagement in theory building in the Patterns class. It was their synergy that led her to use the reference model as a springboard for ontological innovation.

Conclusion

In this paper, we sketched the theoretical genesis of the ontological innovation of meta-theoretic competence (MTC). In particular, a seven-step trajectory from the initial look at the data to theory-in-progress captures one learning scientist's process of theory building, focusing particularly on unpacking the process of using reference models in the process of ontological innovation. We hope that our work in elaborating how a particular theoretical move functioned will help other researchers in the learning sciences recognize similar moves in their work, and to unpack processes of theorizing, for both newcomers and oldtimers in the field.

References

Abrahamson, D., Nathan, M. J., Williams-Pierce, C., Walkington, C., Ottmar, E. R., Soto, H., & Alibali, M. W. (2020, August). The future of embodied design for mathematics teaching and learning. In *Frontiers in Education* (Vol. 5, p. 147). Frontiers.

Adiredja, A. P. (2014). *Leveraging Students' Intuitive Knowledge About the Formal Definition of a Limit*. University of California, Berkeley.

Adiredja, A. P., & Zandieh, M. (2020). The lived experience of linear algebra: A counter-story about women of color in mathematics. *Educational Studies in Mathematics*, *104*, 239-260.

Bakker, A. (2018). *What is design research in education?* Routledge.

Barth-Cohen, L. A., & Braden, S. K. (2021). Unpacking the Complexity in Learning to Observe in Field Geology. *Cognition and Instruction*, 1-33.

Brown, A. L. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *The journal of the learning sciences*, *2*(2), 141-178.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational researcher*, *32*(1), 9-13.

Collins, A., & Ferguson, W. (1993). Epistemic forms and epistemic games: Structures and strategies to guide inquiry. *Educational psychologist*, *28*(1), 25-42.

diSessa, A. A. (1991). If we want to get ahead, we should get some theories. In *Proceedings of the 13th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 220-239).

diSessa, A. A. (1993). Toward an epistemology of physics. *Cognition and instruction*, *10*(2-3), 105-225.

diSessa, A. A. (2004). Metarepresentation: Native competence and targets for instruction. *Cognition and instruction*, *22*(3), 293-331.

diSessa, A. A., & Cobb, P. (2004). Ontological innovation and the role of theory in design experiments. *The journal of the learning sciences*, *13*(1), 77-103

diSessa, A. A., & Sherin, B. L. (1998). What changes in conceptual change? *International journal of science education*, *20*(10), 1155-1191.
diSessa, A. A., & Wagner, J. F. (2005). What coordination has to say about transfer. *Transfer of learning from a modern multi-disciplinary perspective*, 121-154.
diSessa, A. A., Sherin, B., & Levin, M. (2016). Knowledge analysis: An introduction. In *A. diSessa, M. Levin, & N. Brown (Eds.), Knowledge and interaction: A synthetic agenda for the learning sciences*, (pp. 30-71). Routledge.
diSessa, A. A., & Levin, M. (2021). Processes of Building Theories of Learning: Three Contrasting Cases. In O. Levrini, G. Tasquier, T. Amin, L. Branchetti, & M. Levin (Eds.) *Engaging with Contemporary Challenges through Science Education Research* (pp. 225-236). Springer, Cham.
Glaser, B. G., & Strauss, A. L. (2017/1967). *Discovery of grounded theory: Strategies for qualitative research*. Routledge.
Gupta, A., Elby, A., & Danielak, B. A. (2018). Exploring the entanglement of personal epistemologies and emotions in students' thinking. *Physical Review Physics Education Research*, *14*(1), 010129.
Hall, R., & Stevens, R. (2016). Interaction analysis approaches to knowledge in use. In A. A. diSessa, M. Levin & N. Brown (Eds.) *Knowledge and interaction* (pp. 88-124). Routledge.
Hammer, D., & Elby, A. (2002). On the form of a personal epistemology. *Personal epistemology: The psychology of beliefs about knowledge and knowing*, *169190*.
Izsak, A. (2000). Inscribing the winch: Mechanisms by which students develop knowledge structures for representing the physical world with algebra. *The Journal of the Learning Sciences*, *9*(1), 31-74.
Izsak, A. (2005). " You have to count the squares": applying knowledge in pieces to learning of rectangular area. *The Journal of the Learning Sciences*, *14*(3), 361-403.
Izsák, A., Beckmann, S., & Stark, J. (2021). Seeking Coherence in the Multiplicative Conceptual Field: A Knowledge-in-Pieces Account. *Cognition and Instruction*, 1-46.
Kapon, S., & diSessa, A. A. (2012). Reasoning through instructional analogies. *Cognition and Instruction*, *30*(3), 261-310.
Levin, M. (2018). Conceptual and procedural knowledge during strategy construction: A complex knowledge systems perspective. *Cognition and Instruction*, *36*(3), 247-278.
Levin, M. & diSessa, A. A. (2016). "Seeing" as Complex, Coordinated Performance: A Coordination Class Theory Lens on Disciplined Perception. In A. A. diSessa, M. Levin, & N.J.S. Brown (Eds.) *Knowledge and Interaction* (pp. 207-227). Routledge.
Lewis, C. M. (2010, March). How programming environment shapes perception, learning and goals: logo vs. scratch. In *Proceedings of the 41st ACM technical symposium on Computer science education* (pp. 346-350).
Nasir, N. I. S., Lee, C. D., Pea, R., & McKinney de Royston, M. (2021). Rethinking Learning: What the Interdisciplinary Science Tells Us. *Educational Researcher*, 0013189X211047251.
Nathan, M. J. (2021). *Foundations of embodied learning: A paradigm for education*. Routledge.
Parnafes, O., Hammer, D., Louca, L., Sherin, B., Lee, V., Krakowski, M., diSessa, A. A., and Edelson, D. (2008). How to study learning processes? Reflection on methods for fine-grain data analysis.
Parnafes, O. & diSessa, A.A. (2013). Microgenetic learning analysis: A methodology for studying knowledge in transition. *Human Development*, *56*(1), 5-37.
Philip, T. M. (2011). An "ideology in pieces" approach to studying change in teachers' sensemaking about race, racism, and racial justice. *Cognition and Instruction*, *29*(3), 297-329.
Roschelle, J. (2021). Seeing Learning Sciences Research as Modeling. In de Vries, E., Hod, Y., & Ahn, J. (Eds.), Proceedings of the 15th International Conference of the Learning Sciences - ICLS 2021. (pp. 737-740). Bochum, Germany: International Society of the Learning Sciences.
Sandoval, W. (2014). Conjecture mapping: An approach to systematic educational design research. *Journal of the learning sciences*, *23*(1), 18-36.
Sherin, B. L. (2001). How students understand physics equations. *Cognition and Instruction*, *19*(4), 479-541.
Swanson, H. (2022). *Meta-theoretic competence demonstrated by students in the Patterns class.* Proceedings of the 16th International Conference of the Learning Sciences (ICLS 2022). Hiroshima, Japan: International Society of the Learning Sciences.

Acknowledgements
This work was supported by the National Science Foundation (1842375).