

## Meta-Theoretic Competence Demonstrated by Students in the Patterns Class

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**Abstract:** Engaging students in theory building practices is an important focus of science education. This study aims to identify students' resources for engaging in scientific theory building. More specifically, it examines the resources 8th grade students brought to their engagement in theory building in a science elective course called the Patterns class. The Patterns class engaged students in a simplified version of theory building, which involved iterative cycles of articulation, evaluation, and refinement of pattern theories. Pattern theories were written descriptions of general patterns in behavior that could be found in examples across domains, such as threshold and equilibration. Approaching video data and student work through a microanalytic lens, resources were identified belonging to the articulation, evaluation, and refinement of pattern theories across dimensions of deeper structure, generality, domain of application, and causal relationships. In identifying theory building resources, the study introduces a new category of knowledge: meta-theoretic competence.

### Introduction

In recent years, science education has shifted from teaching students the content and skills of science separately, to helping students learn both content and skills in an integrated way, through participation in science practices (Ford & Fordham, 2006). The construction of scientific theory is a central activity of science. Students should therefore have opportunities to engage in theory building in the science classroom and develop skills for participating in theory building practices. In alignment with this perspective, the U.S. Next Generation Science Standards include theory building practices such as modeling and explanation in their learning objectives for K-12 science education (NRC, 2012). Many research programs have investigated the impact of explanation- and model-based instruction on student science learning (Manz, 2012; Passmore & Stewart, 2002). A number of approaches have used explanation or modeling activities as a way of eliciting and building on students' ideas, in alignment with constructivist ideals (diSessa, 1995; Wilensky & Reisman, 2006). Studies to date have primarily focused on developing students' understanding of science concepts by refining their thinking through theory building activities. The present research is complementary to this work, having a long-term goal of designing theory building activities that engage and refine students' resources for theory building. A first step toward this goal is beginning to identify such resources.

There is a rich tradition of research examining the continuities that connect novices and experts, rather than the differences that divide them (Warren et al., 2001). Work within this paradigm has urged education researchers to take an anti-deficit perspective of learners and view them as bringing rich intuitive and culturally-learned funds of knowledge to their learning in and out of school (Adiredja, 2019; Moll, et al., 1992). Leaders in the field have challenged educators to find ways of honoring and leveraging the resources children bring to their learning, particularly with respect to children from non-dominant backgrounds, whose ways of knowing and making sense of the world have historically been marginalized by classroom instruction (Bang, et al., 2012). Research within this tradition has uncovered resources for understanding science topics from the elementary to the undergraduate level, as well as resources for productive engagement in scientific discourse (Hammer, 2000; Hudicourt-Barnes, 2003; Rosebery et al., 2010). The present work aims to make a contribution to this tradition of research by identifying resources students have for scientific theory building. In achieving this goal, it presents a new category of knowledge: meta-theoretic competence.

### Theoretical Foundations

This work is motivated by an epistemological perspective called knowledge in pieces (KiP; diSessa, 1993). KiP grew out of Piagetian constructivism and shares with it a focus on the role of prior knowledge in learning. In addition to constructivism, KiP has roots in cognitive modeling, sharing with it the goal of building a theoretical framework for modeling knowledge and learning. With its attention to technical precision, KiP offers a detailed account of the conjecture that new knowledge is constructed by the learner through engaging their prior knowledge. KiP models an individual's knowledge as a complex system of a diverse array of knowledge elements. The knowledge system of the novice is characterized by loosely connected networks of elements. This means that networks of knowledge drawn on by the novice to explain a phenomenon shift from context to context, responding

to context-specific features of the phenomenon. For example, a physics novice may draw on one set of knowledge elements to explain the orbit of the moon around the Earth and a different set to explain the arc of a tossed ball. The expert, by contrast, would draw on the same network of knowledge to explain both trajectories. KiP models the transition from novice to expert, or learning in general, as a process of reorganization and refinement of the knowledge system.

With this picture of learning, KiP views prior knowledge elements as resources for learning, rather than misconceptions that need to be replaced (McClosky, 1982). Instruction is therefore designed to help students engage and refine their existing ways of thinking and making sense of the world. Researchers working within the KiP paradigm have identified a number of resources that may be seeds for more sophisticated knowledge, practices, and epistemologies. The identification of productive ideas, moves, and perspectives demonstrated by students in both lab and classroom settings has led to the development of a number of new theoretical categories of knowledge.

For example, *phenomenological primitives* (p-prims) are intuitive knowledge elements that guide a learner's intuitive sense of mechanism (diSessa, 1993). *Coordination classes* are larger knowledge structures used in extracting information (like velocity or force) from observable data in the world (diSessa & Sherin, 1998). These categories of knowledge include resources for the development of conceptual understanding in science. Other categories of knowledge include resources for the development of skills for participation in scientific practices. For example, *meta-representational competence* (MRC) includes resources for constructing, critiquing, and making sense of representations (diSessa, 2004). Yet other categories of knowledge include resources for understanding the nature of science. For example, *epistemological resources* include resources for understanding the nature of knowledge and its origins, epistemological activities, and positions one might take towards knowledge (Hammer & Elby, 2002).

The present work uses meta-representational competence as a reference model for organizing resources for scientific theory building. To make the link to this model explicit, I call this category of knowledge *meta-theoretic competence* (MTC). Meta-representational competence features knowledge that transcends the production and use of representations (diSessa, 2004). It includes skills for the invention of new representations, for their critique, and for their explanation. Resources for representation invention include the ability to draw, and sensitivity to representational possibilities of visual elements (e.g., color, length, width). Resources for the critique of representations include criteria such as completeness, precision and parsimony. The resources belonging to MRC are assumed to be "gradually developed through cultural practices in and out of school" (diSessa, 2004, p.294). In the next section, I describe the empirical context for my work and my approach to analyzing data, which led to my development of meta-theoretic competence as a category of knowledge resources.

## Methods

This work is situated in the context of a larger research project, which was aimed at the design of a middle school course on topics in the domain of dynamical systems theory. DST is a highly mathematical science, which uses differential and difference equations to model systems that change over time. The course guided students' development of descriptions of general patterns of change, which could be found in phenomena across domains. Such patterns include threshold, equilibration, exponential growth, and oscillation. Threshold, for example, can be seen in the tipping point of a tower of blocks and the limit of a person's patience. Many of the descriptions of these patterns, which the students wrote, were essentially conceptual versions of the mathematical equations DST uses to model phenomena. Our research team called students' written descriptions *pattern theories* and engaged students in an iterative process of pattern theory building and refinement. We called the course the Patterns class, because of its focus on exploring and describing patterns. The goal of the present work is to understand the productive resources students brought to their engagement in theory building in the context of the Patterns class.

The Patterns class was taught in a public middle school in an economically depressed neighborhood of a large city on the West coast of the United States. The school was selected based on the willingness of the 8<sup>th</sup> grade science teacher to share her elective period with the research team. She had traditionally used the elective as a science enrichment course for students who were proficient on tests of basic skills in English and math. The author taught the course, having been a high school science teacher for six years before transitioning to research. The class met on Monday, Tuesday, and Thursday mornings for 40 minutes, totaling 52 hours over the school year. Twenty-one 8<sup>th</sup> grade students (11 girls and 10 boys) participated in the course. Eighteen of the students had immigrated to the U.S. from Mexico and Central America. Two students identified as African American and one as Bosnian American. The majority of students attending the school were designated as English Language Learners and from low-income households. The students were selected on the basis of their availability and willingness to participate in the experimental curriculum.

## Patterns class design

The course was designed to engage students in a simplified version of scientific theory building through an iterative process of theory articulation, evaluation, and refinement. Through this process, students constructed and refined theories for patterns including threshold, equilibration, exponential growth, and oscillation. Each unit opened with the exploration of two pattern examples, followed by students' articulation of their first draft pattern theories. The students then evaluated their theories against a third example and revised their thinking, producing second draft pattern theories. They generated additional examples and evaluated their theories against those, producing third and final draft pattern theories. In addition to mirroring a simplified scientific theory building process, the sequence was meant to help students articulate and iteratively refine their thinking and their theories. Below, I briefly unpack the first two pattern units, to give the reader a sense for the Patterns class and to provide a basic chronological reference for the narratives I provide to illustrate the elements of meta-theoretic competence.

### Threshold unit

The threshold unit was designed with a pattern benchmark in mind. The threshold benchmark consisted of a *pre-phase*, *limit*, and *post-phase*. During the pre-phase, a system parameter is varied until it reaches a limit. When the parameter exceeds the limit, the system transitions to a post-phase, which is markedly different from the pre-phase.

The threshold unit began with the exploration of two examples: the addition of coins to a cup hung over a spaghetti bridge until the bridge broke, and the addition of drops of water to a coin until the water flowed onto the table. The students explored these examples in the context of small group team challenges, which were each allocated a single class period. Following their exploration of these examples, the students individually wrote their first draft pattern theories. Their theory building was guided by a prompt, which focused them on describing the pattern in behavior common to both examples: "Tell the story of both behaviors so that someone listening to your story would agree that you are talking about either one of the behaviors, but they wouldn't know for sure which one you were talking about." This activity was allocated about half a class period.

The students then evaluated their pattern theories against a third example: adding salt to a cup of water until a submerged egg floated to the surface. They explored this example in small groups in the context of an investigation, which was allocated a single class period. Students were asked to compare the egg example with the spaghetti bridge and drops-on-a-coin examples and record their thoughts in writing. The teacher collected their responses and presented the class with a representative sample of both their example comparisons and written pattern theories, facilitating a whole-class reflection during which students compared pattern theories with each other and with examples. The reflection discussion was given a single class period. The students were then invited to revise their pattern theories and write second drafts. This activity was allocated about 10 minutes and took place following the reflection discussion.

The students were then invited to generate examples from their own lives, which followed the pattern. Students worked alone to generate initial lists and then in small groups to create posters showcasing their favorite examples. They hung their posters around the classroom and participated in a gallery walk, considering their classmates' work. During the walk, they applied sticky notes to posters to identify examples they agreed with, disagreed with, or didn't understand. The teacher selected two contentious examples to feature in a whole-class debate (e.g., "bothering someone until they burst" and "getting a haircut"). Before the debate, she gave the students time both individually and in small groups to prepare arguments for why these examples did or did not follow the pattern. She then asked students to consider one example at a time, offering points to support or challenge the example. The students were invited to revise their pattern theories a final time and write third drafts. This activity was allocated about 10 minutes and took place following the debate.

### Equilibration unit

The equilibration unit was designed with a pattern benchmark in mind. The equilibration benchmark was a pattern of *difference drives rate*. An equilibrating entity moves towards equilibrium at a rate proportional to the difference between its current state and its equilibrium state. A pattern of *difference drives rate* can be seen in both thermal equilibration and the calming of emotions over time.

The equilibration unit began with the exploration of two examples of thermal equilibration: the warming of cold milk to room temperature and the cooling of hot tea to room temperature. The students explored these examples in the context of small group investigations. The investigations took the form of traditional science labs and were each allocated a single class period. They were each preceded by activities in which the students created and presented hypotheses of how the temperature would change over time. They were each followed by activities guiding data interpretation and activities focused on the generation and discussion of potential causes for the fast-

then-slow temperature change. Following their exploration of these examples, the students were given half a class period to write their first draft pattern theories.

The students then evaluated their pattern theories against a third example: the diffusion of beans through a semi-partitioned box as they shook the box back and forth. They explored this example in small groups in the context of an investigation, which was allocated a single class period. They were asked to compare the bean example with the warming and cooling examples and record their thoughts in writing. The teacher collected their responses and presented the class with a representative sample of both their example comparisons and written pattern theories, facilitating a whole-class reflection during which students compared pattern theories and examples. The reflection discussion was given a single class period. The last 10 minutes of the period were reserved for the students to revise their pattern theories and write second drafts.

As in the threshold unit, the students were invited to generate examples from their own lives, which followed the pattern. They worked alone to generate ideas and then in small groups to create posters showcasing their examples. They hung the posters around the classroom and participated in a gallery walk, using sticky notes as they had in the threshold unit. The teacher selected a contentious example to feature in a whole-class debate (emotions calming). She gave the students time to prepare their thoughts on the example. During the debate, she asked students to either offer points in support of *emotions calming* as an example of the pattern, or points to challenge it. Following the debate, the students were invited to revise their pattern theories and write third and final drafts.

## Data collection

Data were collected in the form of video footage, student work, and teacher reflections. Each class period was videotaped using two cameras. One camera was positioned at the front of the classroom and captured the students sitting at their tables in groups of 4. The other camera was positioned at the back of the classroom and captured the activity at the front board when the teacher or student presenters were speaking. When the class engaged in small group or individual work, this camera was repositioned to capture the work of one group. Student work was collected at the end of each class period. The work was scanned and returned to the students the next day to keep in their course portfolios. As the teacher, I recorded reflections at the end of each lesson, documenting things I had noticed in terms of student thinking and engagement, with respect to the course design.

## Data analysis

The broader goal of the present study was to understand the nature of student theory building in the Patterns Class. I approached this goal through a KiP lens, expecting to find evidence of students' productive engagement in theory building. I analyzed student work using a microanalytic approach characteristic of knowledge analysis (diSessa, Sherin, & Levin, 2016). I examined students' written theory drafts for theory-like characteristics, and their notes from activities for evidence of their thinking between theory drafts. I reviewed the written teaching reflections, looking for class or small group discussions related to students' articulation and refinement of pattern theories. The teaching reflections helped me identify video footage to review, transcribe, and analyze. While watching the video footage, I noted on the transcript where I saw evidence of productive engagement in theory building, naming and briefly characterizing what looked like ideas, moves, and aesthetics that seemed productive with respect to students' theory building. I collected the elements I had named in a separate list.

It became apparent that the list of elements were student resources for theory building and the focus of my inquiry narrowed to identifying the productive resources students brought to their theory building in the Patterns class. The list was analogous to a collection of resources for engaging in another set of scientific practices, which already existed as a category of knowledge in the KiP literature. This category of knowledge was meta-representational competence (MRC; diSessa, 2004). My collection of resources for theory building was parallel to the collection of resources belonging to MRC, in several ways. Both are capacities for engaging in multiple dimensions of a realm of scientific practice (including generation and critique of scientific artifacts), and both include capacities gained from interactions with others in and out of school. Because of this analogy, I decided to use MRC as a reference model and named my category of resources *meta-theoretic competence* (MTC).

I reviewed the list, which I had thus far compiled, and combined resources that seemed similar and clarified distinctions between resources that seemed similar but were different in important ways. In sum, I engaged in an epistemic game of "list making" (Collins & Ferguson, 1993). I moved back and forth between my list and the data, refining the list so that it more accurately captured the productive things students were doing with respect to theory building. When I arrived at a final list, I described each resource and connected it with a particular example from the data. In working out the organization of the items on the list, it became apparent that there were two dimensions according to which the items on the list should be organized. These dimensions were stages of theory building (e.g., articulating, evaluating, and refining theories) and pattern-theory-specific elements

(e.g., deeper structure, generality, domain of application, and causal relationships). Because items on the list were connected with elements from both dimensions, I transferred my epistemic form from a list to a cross product or table (Collins & Ferguson, 1993).

## Findings

My analysis of student engagement in the Patterns class produced a table of resources for meta-theoretic competence. The table outlines resources for three stages of theory building (articulating, evaluating, and refining pattern theories) across four characteristics of pattern theories (deeper structure, generality, domain of application, and causal relationships). Table 1, below, is meant to give the reader a sense for the larger ontology of meta-theoretic competence. Due to space limitations, I list only one resource per cell.

Table 1. Theory building resources demonstrated by students in the Patterns class

Stages/Pattern Characteristics	Deeper Structure	Generality	Domain of Application	Causal Relationships
Articulating	Articulating the pattern in behavior instantiated by examples	Using general language to describe a pattern in behavior	Generating examples from physical and psychosocial domains	Articulating intuitions about causal relationships underlying behavior
Evaluating	Comparing examples based on similarities and differences	Questioning the generality of the language used	Challenging and defending the fit between examples and pattern theories	Challenging and defending the logic of intuitions
Refining	Removing surface features from the pattern theory	Removing context-specific features to make the pattern theory applicable to more examples	*There was no opportunity in the course for students to demonstrate resources for this category	Responding to challenges by unpacking or modifying ideas

I unpack the list below, illustrating each category of resource with narrative from the class, drawing on either video transcript or student work. In the narrative, I highlight the particular resource using italics.

### Deeper structure

An ability to look beyond a phenomenon's surface features and articulate its deeper structure is a hallmark of expertise in science and fundamental to the construction of scientific theories (Chi, Feltovich, & Glaser, 1981). Deeper structure can take different forms, from a mathematical relationship between system variables, to a mechanism driving system behavior. In the Patterns class, a theory attended to deeper structure if it described the *pattern in behavior* common to multiple examples, rather than their common surface features.

#### Articulating deeper structure

In the threshold unit, students were asked to individually write their first draft pattern theory after exploring two examples: adding coins to a spaghetti bridge until it snapped and adding drops of water to the surface of a coin until the water overflowed. Their theory building was guided by a prompt, which focused them on describing the *pattern in behavior* common to both examples: "*Tell the story of both behaviors* so that someone listening to your story would agree that you are talking about either one of the behaviors, but they wouldn't know for sure which one you were talking about." Some students produced pattern theories that focused on common surface features, for example: "Both involved household items, needed techniques in order to do both of these experiments, both took patience." Many of the students, however, *articulated patterns in behavior* common to both examples, for example: "Both of these patterns involve adding things to objects until they can't hold those objects anymore."

#### Evaluating deeper structure

Students were asked to evaluate their first draft pattern theory against a third example: adding salt to a cup of water until a submerged egg floated to its surface. Two examples of threshold were observed in this example by the students: the floating of the egg in response to the addition of salt, and the saturation of the water with salt. This led to two different conceptualizations of the pattern: one of threshold as *tipping point*, and one of threshold as *maximum capacity*. Following their exploration of the example, the students individually responded to prompts asking them to compare the new example with the initial two examples. Comparing the saturation of the water with the spaghetti bridge, Patricia *identified a similarity in behavior*, writing: “We had to see how many drops an object would hold.” Another student, Selena, compared the floating egg with the spaghetti bridge. She *identified a similarity in behavior*, writing: “You added more objects in it.” She also *identified a difference in behavior*, writing: “We were seeing if it floats and in the spaghetti bridge we were seeing how many pennies you could put before it breaks.”

### Refining deeper structure

Following the egg investigation, the teacher led the students in a discussion where they compared the examples they had thus far explored as well as their first theory drafts. The students were then invited to revise their thinking and write second draft pattern theories. Patricia’s initial theory of the pattern had focused on the common material and social features of the spaghetti bridge and drops-on-a-coin examples: “Both used pennies, had to have a special technique.” Her second theory draft characterized the pattern as: “They all involve adding something to get a *reaction* out of it. It mostly resulted in something overflowing up other objects.” Notably, she refined her theory by *removing surface features* and focusing instead on the deeper pattern in behavior underlying examples.

## Generality

The power of a scientific theory, in part, depends on its range of applicability, and therefore its level of generality (Atkins, 2010). In the Patterns class, a theory was considered general if it *used language that did not tie it to any particular example*.

### Articulating a general theory

The students practiced tuning their theory’s level of generality during each pattern unit when they responded to the prompt by telling “the story of both behaviors so that someone listening to your story would agree that you are *talking about either one of the behaviors, but they wouldn’t know for sure which one you were talking about*.” Students’ pattern theories were considered general when they were written in language that did not tie them to a particular example. Their initial drafts in the threshold unit included theories that described the pattern in context of specific examples, for example: “We used pennies, we put the pennies in a container, we counted, we did it again.” The students’ later drafts tended to be context-free, *using general language to describe the pattern in behavior*, for example: “Adding something to something until it changes.”

### Evaluating the generality of a theory

Students were shown their peers’ theory drafts before revising their theories. The teacher projected anonymous examples of students’ work and asked the class to compare their theories. During this discussion, Alvaro problematized the name one group had given the pattern – “The Break” – saying: “Actually, not everything had to do with something breaking. Because you know the one where we squirted the drop of water onto the pennies? It didn’t break, it spilled...” Alvaro pointed out the inadequacy of the word break, on the grounds that it didn’t apply to all examples. In doing this, he demonstrated a resource for evaluating a theory’s generality, specifically by *questioning the generality of the language used*.

### Refining the generality of a theory

Students refined their pattern theories’ level of generality. For example, in the threshold unit, one student’s initial theory read: “We used pennies and with those pennies only a certain amount of weight had to be held before it interrupted the experiment.” The pattern is described in the context of the spaghetti bridge investigation and is therefore not general. For their second draft of the pattern theory, the same student wrote: “You need to add something on an object until it breaks or you can’t fit any more.” The student *removed context-specific features* from their theory, making it applicable to a broad range of phenomena.

## Domain of application

A scientific theory is applicable to a bounded regime (Suppe, 1972). In the Patterns class, students specified their theory’s domain of application by *listing examples* that followed the pattern.

### Articulating the domain of application

Once students had written their second draft pattern theories, they were invited to generate a list of their own examples. For the threshold unit, students *generated psychosocial examples* such as “adding more sadness until you cry,” and “bothering someone until they burst.” They *generated physical examples* such as “filling up a water balloon,” “pulling gum until it snaps,” and “getting a haircut.”

### Evaluating the domain of application

From the examples the students generated, the teacher identified two to feature in a whole-class debate: “bothering someone until they burst,” and “getting a haircut.” During the debate, students *challenged the fit between examples and theories* by attending to mappings between the two. For example, Martin challenged the example of bothering someone until they burst, saying: “There’s not really a maximum, like you could keep annoying them... ‘cause after they get burst you could still keep annoying them, it isn’t, it won’t, it’s not going to get fulfilled up on anything.” Martin viewed the pattern as maximum capacity, and for him, there was no maximum limit to bothering someone.

## Causal relationships

A chief application of scientific theory is the explanation of observed phenomena (Hempel & Oppenheim, 1948). In the Patterns class, a theory was seen as articulating causal relationships if it moved beyond describing the pattern in behavior to *include a causal explanation* for it.

### Articulating causal relationships

The students were encouraged to think about the causal driver underlying a pattern in the second unit of the course, which focused on equilibration. They participated in a whole-class discussion with the aim of developing a causal explanation for an example where a glass of cold milk warmed to room temperature fast-then-slow. Before the discussion, the teacher asked the students to write down their initial explanations. She then seeded the discussion by reading the students’ explanations aloud, leaving them anonymous. The explanations included ideas about the milk slowing down to reach room temperature, the physical apparatus used to conduct the experiment, and the milk adjusting to room temperature. In generating these causal explanations, students demonstrated resources for articulating causal explanations in the form of *intuitions about causal relationships*.

### Evaluating causal relationships

Having read all of the ideas, the teacher focused the students on one idea in particular: “Because it was getting to room temperature at the end, so it was slowing down. It’s like a race, when you’re getting to the destination you start to slow down.” The students debated the idea, demonstrating resources for challenging and defending its logic.

Leo: Why would you slow down when you’re about to finish a race? It doesn't make sense.

Alvaro: Say there’s a wall. Are you going to run straight into it, Leo?

Leo: Well, I’m not gonna go slower though, because then I’ll lose.

Alvaro: Like no-no-no-no-no! Like, say you're winning ‘cause you’re going as fast as you can, then when you’re gonna reach the wall, don't you start to like kinda? [stomps feet on ground slowly]

Leo *challenged the explanation on the grounds of its logic*: who slows down at the end of a race? Alvaro *defended the explanation by modifying the race analogy* to make it logical, turning the general scenario of a race into a race that specifically ended at a wall, which he subsequently identified with room temperature.

### Refining causal relationships

Through their discussion, the students refined a combination of several ideas to produce an explanation that was conceptually similar to Newton’s law of heating: “That it has lots to cover so it just does it fast and then it starts slowing down because it needs to reach, it’s almost reaching its max.” The explanation was developed through students’ modification of ideas in response to problems pointed out by their peers. In this way, students demonstrated resources for refining causal explanations, such as *addressing challenges by modifying ideas*.

## Discussion

This paper presented a collection of resources for theory building, which were demonstrated by the 8th grade students who participated in the Patterns class. The collection of resources is only preliminary. It is assumed that other students participating in the same course would demonstrate different resources, and that different kinds of theory-building activities (e.g., computational modeling) would elicit different resources. Despite its limited scope, the list of resources for theory building makes an empirical contribution to literature concerned with identifying the productive knowledge students bring to their learning, responding to the call to education researchers to view students from an asset-based perspective. More broadly, the category of knowledge to which the theory-building resources belong is a theoretical contribution to the KiP research program, as an ontological innovation.

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