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When failing generates math anxiety instead of productivity

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Abstract: Experiences of failure and confusion can be intentionally purposed for learning tasks. In the Productive Failure instructional design, students attempt to solve problems before they are formally instructed on the relevant content. However, confusion and the perception of failing can elicit negative emotions in learners, even more so in students who already feel anxious about failure. Although research has demonstrated that failure can lead to math anxiety, and vice versa, the role of math anxiety in instructional designs that leverage moments of failure has not been investigated. This negative emotional cycle may negatively impact some students' learning and interest. We argue that future research would benefit from considering potential tradeoffs between enhanced learning and increased anxiety. We ground our arguments in an exploratory mixed-methods design that combines behavioral data with a task-based interview.

Affect and Math Anxiety in Math Education

The various elements of affect, attitudes, and student beliefs—such as those concerning one's understanding of math, or beliefs concerning success and failure—are increasingly acknowledged to play an important, yet poorly understood, role in math learning. For example, students' affective disposition can be a more critical predictor of entry into STEM subjects than enrollment and grades (Maltese & Tai, 2011).

When experiencing a negative emotion, we may focus on our negative sensations. Psychologically, this is done to find the means of eliminating the negative feeling. Thus, it is not surprising that there is an effect of emotions on learning, and studies have shown that learning is negatively influenced by unpleasant emotions, unless the tasks are simple (attention allocation elsewhere; see Hascher, 2010). Regardless of their valence, emotions can also be beneficial for learning, such as happiness (Kim & Pekrun, 2014), confusion (D'Mello et al., 2014), and even frustration (Liu et al., 2013). On the other hand, boredom and anxiety are associated with reduced learning (Pekrun, Elliot & Maier, 2009), while frustration can be destructive (Kort, Reilly & Picard, 2001).

While uncertainty has been shown to be beneficial for learning, it may also be detrimental (Lamnina & Chase 2019). In short, instructional designs that intentionally generate uncertainty could contribute to either:

(1) negative emotions about knowledge (Sinha 2021): a focus on the uncertainty of the problem, or
(2) negative achievement emotions (Pekrun & Linnenbrink-Garcia, 2014): a focus on personal failure.

Thus, fostering uncertainty could lead to (1) curiosity to learn what the student is (now aware to be) lacking—but may also lead to (2) self-preoccupying thoughts that interfere with learning. The latter may even feed into vicious cycles of anxiety and failure (Pekrun, Elliot & Maier, 2009). Some affective aspects in math learning appear to be rooted so deeply that they are difficult to diminish by instruction (Gomez-Chacon, 2000).

Because there is more to a learning situation than the *local* affect, we also must attend to the *global* affect—the lingering presence of experiences that made the learner the person they are (Gomez-Chacon, 2000). We argue that it is important to also attend to the more global affects, such as *math anxiety*, when designing for uncertainty. Math anxiety, which develops through various educational and non-educational experiences throughout the learner's life, has been described as a feeling of panic, helplessness, paralysis, and mental disorganization that arises when one is solving a mathematical problem or managing numbers (Ashcraft, Kirk & Hopko, 2000). In interviews with math anxious individuals (Sloan, 2010), the most reported causes of math anxiety were: negative experiences with math classes, teaching practices in school, memorization of procedural knowledge, math avoidance—low math achievement, test anxiety—failure, lack of confidence. Thus, pre-existing math difficulties cause or aggravate math anxiety (Núñez-Peña & Suárez-Pelliconi, 2014), and poor math performance can lead to math anxiety. Taken together, math anxiety is not merely a local affect but is better understood as a global affect that influences learners' behavior and cognition. This global affect ought to be considered when designing instruction that generates instances of failure and uncertainty, because this approach could generate or reinforce a student's already existing negative attitudes, beliefs, and feelings towards math.

Productive Failure and Affect

Productive failure (PF; Kapur, 2016; Kapur & Bielaczyc, 2012) is a subset of Problem Solving followed by Instruction designs (PS-I; Loibl, Roll, & Rummel, 2017), that broadly follow the paradigm of preparation for

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future learning (Schwartz & Bransford, 1998). In PF, the problem-solving phase is intentionally designed as an initial generation and exploration phase, which offers students the opportunity to activate existing and intuitive knowledge. Since the problems are based on concepts that have not been formally taught to the students, the problem-solving phase typically generates feelings of uncertainty and moments of failure (e.g., incorrect solutions). In a subsequent instruction phase, the desired concepts are conveyed to the students.

PF designs, like impasse driven learning (Van Lehn, 1988), lead students to an impasse in their learning that forces a transition into "repair" strategies, such as seeking help, to overcome the impasse. Piaget (1964) formulated in his theory of cognitive development that we experience a state of imbalance when encountering information that requires us to develop new schema or modify existing schema. This uncomfortable state of imbalance drives us to the assimilation of new schema or the adaptation of old schema. Without accommodation, we are left with old schemas. For some, this might be "I am not good in math; I don't know how to do it".

Present Study

Only a few studies investigated the role of affect in PF (e.g., Lamnina & Chase, 2019; Sinha, 2021). To our knowledge, we are the first to investigate math anxiety specifically in a PF design. Math anxiety affects all stages of cognitive math processing (e.g., Ashcraft, Kirk, & Hopko, 1998). Therefore, we hypothesize that students with higher levels of math anxiety will benefit comparatively less from the PF design on learning outcomes.

Method

Participants

Sixty-two students (age range = 18-24) were randomly assigned to one of the conditions (PS-I or "Direct Instruction followed by Problem solving - I-PS"). Two students dropped out due to their high level of math anxiety. Those two students took longest on the task and vocalized their perceived difficulties and feelings towards the situation. The first author took notes of these comments and conversations and translated them to English. All participants were naïve to the goals of the experiment and provided written informed consent. The study has been approved by the local Ethics Committee following the Declaration of Helsinki.

Procedure and Materials

A classical PF design (Kapur, 2014) was used along with questionnaires about different cognitive and affective mechanisms such as knowledge gap awareness, state curiosity, cognitive load, learning goal orientation, and attitudes towards mistakes (Sinha & Kapur, 2021). In addition to that and relevant to the present study, math anxiety was measured with the Abbreviated Math Anxiety Scale (AMAS; Hopko et al., 2003). The participants responded using a 5-point Likert-scale, ranging from 1 (low anxiety) to 5 (high anxiety).

Results and discussion

Quantitative results

An ANCOVA was performed to investigate whether the conditions would differ in the posttest scores. The results are shown in Table 1. Contrary to expectations from previous research, the students in the PS-I condition (M = 19.37) did not significantly outperform those in the I-PS condition (M = 19.03). Moreover, there were no differences on any other cognitive and affective measurements. However, the impact of math anxiety—as a covariate in the analysis—was highly significant and could explain why the students in the PS-I condition did not perform better on the posttest. This supports our hypothesis that math anxious students—who were more prominent in the PS-I condition—do not benefit as much as non-math anxious students from the PF design.

Table 1
ANCOVA for Achievement by Instruction Type, Attitudes and Feelings

		Achievement		
Type of Instruction		**Mean**	**SD**	**N**
I-PS		19.03	4.00	30
PS-I		19.37	3.45	32
Source	**SS**	**MS**	**F**	**η^2_p**
Condition	12.59	12.59	1.17	0.02
Math Anxiety	121.30	121.30	11.31*	0.16
Learning Goal Orientation	4.12	4.12	0.38	0.01

Attitude Towards Mistakes	1.76	1.76	0.16	0.00
Knowledge Gap Awareness	36.97	36.97	3.45	0.05
State Curiosity	3.95	3.95	0.4	0.01
Germane Cognitive Load	0.76	0.76	0.07	0.00
Residuals	589.35	10.73		

Note. Levene's Test for Equality of Variances $F(1, 60) = 0.031, p = .9$; *p < .001

Qualitative results and discussion

The quantitative results do not capture the experience of the excluded participant Belinda (name changed; highest AMAS score in the sample: 43). The inclusion of interview notes illustrates the challenges faced by math-anxious individuals when dealing with designs that intentionally introduce feelings of uncertainty and failure.

> Belinda: [looking at first author after reading the problem] "Oh, I don't know how to do this!"
> First author: "It's okay, it's about learning something new, the solution doesn't have to be right."
> Belinda: "Okay, but this is math, and I am *not* good in math." […]
> [solving the task shows motor restlessness, changes sitting position, fidgets, sighs heavily]
> Belinda: "Can you please give me clues? I don't even know what to do. […] If I don't get an example of how to solve [a math problem] or an explanation I don't know how to solve it."
> First author: "[…] It's not about doing it correctly, just about trying. […]"
> Belinda: "Okay, I'll try. If I do this [shows notes] is it going in the right direction?"
> First author: "See, you're coming up with something without being instructed."
> The problem-solving phase ends, and first author explains the next step of the instruction phase.
> Belinda: "Oh no, now I will see how wrong I was."
> First author: "You'll see a standard way to solve the problem. You're not wrong but creative."
> Belinda: "Erm… So… my nervousness has increased quite a bit over time, and…I'm actually very nervous. Maybe also a bit afraid… afraid that I will fail. I am not sure if I will recall…"
> First author: "[…] it is okay if you don't recall the formula, maybe you recall some parts of it or comprehend what the formula said. That's all fine and you're already learning. […] even if you wouldn't recall the formula, you'll be okay… you can stop whenever you want."
> Belinda: "Okay, thank you... [sighs] …" [reads the questions and attempts to solve the tasks, sighs heavily and buries her head into her hands, but continues, persevering for 20 minutes]
> Belinda: "I can't do this. I am trying but I'm frustrated, I'm also afraid to fail, it makes me think of my fear instead of focusing on the questions. Can I stop please?"
> [Task is stopped. Conversation continues with support for anxiety, not relevant to task]

The case of Belinda highlights how this instructional strategy had a severe impact on one participants' math anxiety during the learning process, leading to them dropping out in the posttest.

Conclusion and Future Directions

While PF designs tend to produce positive effects on student understanding, they also generate moments of failure and feelings of uncertainty. This is not a criticism *per se*: after all, learning math *is* difficult. However, the concern is that certain learners might experience increased math anxiety and more negative opinions regarding their abilities and knowledge. Even if learning can be increased with such designs, do we not also court the risk of losing potential STEM students? As Maltese and Tai (2011) suggests, it may be possible to simultaneously *increase* students' math achievement yet *decrease* their interest in majoring in STEM fields.

Further research should be done to determine whether the present results can be generalized. Future experimental studies should contrast math anxious and non-math anxious students in a PF design (i.e., a 2x2 design, PS-I vs I-PS, math-anxious vs non-anxious) to investigate if math anxiety leads to lower posttest scores. Nonetheless, the present results offer support for our central argument that math anxiety plays a significant and thus-far unexplored role in instructional designs that generate moments of uncertainty and failure.

Course instructors and teachers should be aware of these findings, such that appropriate help and support can be provided to. In an ideal setting, students can experience curiosity and benefit from such designs, yet "ideal" is not something we can intentionally design for. Increased attention to affect in the learning of math, and particularly to math anxiety, would bring us closer to our shared goal of "educational innovation for all."

Author Contributions

VG: Project Administration, Conceptualization, Methodology, Writing- Original draft. DT: Writing- Reviewing and Editing, Resources. CF: Investigation, Formal Analysis.

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