Turning to Experience Negative Signs as Operations

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Abstract: Conceiving of negative sign notation as an operation is crucial for algebra. Thus, this study compared student learning with two integer models to explore how physical motions influenced conceptions of this notation. Preliminary results with 70 fifth-grade students suggested that physically turning the opposite direction better represented this notation as an opposite operation than moving objects with a chip model (p=0.001). These findings contribute to mounting evidence that how humans physically move influences learning.

Introduction

Internationally, it is well known that middle and high school students find operations with negative numbers difficult. Although research has studied representations of number, such as number lines (e.g., Siegler, 2016), such representations are just one part of a model. I define models as the patterns of physical motions and words used with particular representations that are intended to foster learning. Multiple integer models, such as chip and number line models are used in classrooms (van de Walle, et al., 2010). Comparisons of how specific models support or interfere with learning aspects of integer operations, however, are just beginning (Nurnberger-Haag, In Press; Tsang, Blair, Bofferding, & Schwartz, 2015). A crucial aspect of integer understanding that is difficult for students is to think of the symbol "-" as an operation that means "take the opposite of" (Ryan & Williams, 2007; Vlassis, 2008). Although this may seem a narrow research focus, consider that most equations in algebra and calculus require students to coordinate a negative sign as an operation (e.g., –(-2x – 4) or –X, "the opposite of X") as well as part of the structure of a number (e.g., -5; Vlassis, 2008).

Drawing on embodied cognition that how humans move influences how they think (e.g., Antle, 2011; Barsalou, 2008), this study tested a prediction that different models would differentially influence understanding of integer notation. Students who use the walk-it-off number line model physically "turn the opposite direction" for subtraction and negative signs (Nurnberger-Haag, 2007). For example, students stand next to the point -5 on a large number line, turn the opposite direction twice (once for the subtraction sign and once for the negative sign of -2) and walk two, resulting in an answer of -3. Students who use a chip model take away objects to subtract (e.g., students act out -5 - -2 with five white chips then remove two, resulting in -3). Thus, this report addressed the question: Do students who use a "turn the opposite" meaning of the "-" symbol learned with the walk-it-off model interpret a negative sign as an operation better than students who used a chip model.

Method

In a larger study eight classes at a fifth and a sixth-grade site were randomly assigned to learn with either a chip or number line model (Nurnberger-Haag, In Press). The chip model used black and white chips to represent positive and negative numbers respectively. The walk-it-off number line model emphasized the "-" symbol as meaning "turn the opposite direction" (unlike traditional number line models that treat subtraction as backward motion; Nurnberger-Haag, 2007). The researcher instructed all classes in eight parallel lessons differing only due to model. During instruction students only learned to add, subtract, multiply and divide negative numbers and represent opposite numbers (not operations) using the assigned model. To eliminate the possibility that students who memorized notational patterns would be indistinguishable from those who understood the meaning of the negative sign as an operator, opposite operations was assessed as a transfer construct (i.e., students were not taught these types of problems during instruction).

After five phases of piloting with 388 students, a written open-response scenario was finalized to elicit whether students could conceive of the "-" notation as an operation. An abridged description of the item is that "(negative number)" was presented in blue font and "–(negative number)" in green to elicit students to describe how the quantities differed. Post and delayed posttests of the three fifth-grade classes are reported (n=70 students), because no fifth-graders correctly answered at pretest. These responses were coded in terms of whether students offered a non-negative solution, and then integer model groups were compared with a chi-square analysis. Using embodied cognition, qualitative analyses are being conducted to provide insights about the ways and reasons students' physical motions on a number line or ways they moved objects related to their conception of the symbol "-".

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Results

Given the difficulty of the construct, it was expected that few students would successfully answer this transfer item. Overall 7% transferred at posttest and 11% did so on the delayed posttest. Although 1.7 times as many students learned with a chip (n=44) than number line model (n=26), just 2% of students who used chips provided a non-negative solution whereas 15% of students who used the walk-it-off model did so. The chip student who was successful at posttest, regressed at delayed post, so 0% (0/40) of students who had used chips answered correctly at delayed posttest. In contrast, 27% (7/26) of students who learned with the walk-it-off model explained that the solution would be non-negative. A chi-square analysis using Fisher's exact test confirmed this difference was statistically significant (p=0.001) at delayed posttest. At posttest, the significance was close to threshold, but did not meet it (p=0.06). On the poster, student drawings and explanations will show how walk-it-off students used physical motions, arrows, and words like "turn opposite" to reason through this task that was beyond their instructional experience in contrast to students who moved chips and did not express these conceptions of negative signs.

Conclusions

By delayed posttest more than one-fourth of students who learned with the walk-it-off model could conceive of –(negative number) as a positive number, whereas no students who used a chip model did. Although the eighth-grade students in Vlassis' (2008) study had difficulty conceiving of the "-" symbol as an operation and even sometimes confused it with a subtraction sign, a substantial number (for a transfer item) of the fifth-grade students in this study were able to apply their experiences of adding, subtracting, multiplying, and dividing negative numbers to also conceive of the operation "opposite of" without explicit instruction on those problem types. This preliminary analysis suggests that physically turning the opposite direction on a number line in combination with related language "turn the opposite direction" better supports students to extend the meaning of the "-" symbol to an operation than the symbol meanings students act out with a chip model. This finding is consistent with other studies of cognition and learning which found that how humans move to represent ideas matters (Antle, 2011; Nurnberger-Haag, In Press), but extends this to how learners apply their physically constructed understanding to make sense of new ideas. More embodied cognition research is needed to study how each model supports or thwarts the complex meanings students need to develop about integers. Next steps include investigating students who learned with multiple integer models, disentangling effects of the representation from physical motions, and uncovering relationships between the effects of physical motions and the language used to describe them. These future studies could contribute to understanding how human movement relates to cognition and learning.

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