Mapping Student Reasoning in Support of Mathematics Teacher Candidate Digital Learning

Laurie Cavey, Boise State University, lauriecavey@boisestate.edu
Tatia Totorica, Boise State University, tatiatotorica@boisestate.edu
Michele Carney, Boise State University, michelecarney@boisestate.edu
Patrick Lowenthal, Boise State University, patricklowenthal@boisestate.edu
Jason Libberton, Idaho State University, libbjaso@isu.edu

Abstract: Using design-based research, we are developing a series of video-based online learning modules to engage mathematics teacher candidates in analyzing cases of secondary students' mathematical reasoning. We describe our process for developing a student reasoning framework for a task that requires the coordination of two relationally specified quantities, which provides a basis for selecting module content and analysis of candidates' attention to student reasoning as they engage with the online modules.

Introduction

The Video Case Analysis of Student Thinking (VCAST) project entails the design and integration of a series of video-based online learning modules into undergraduate teacher preparation coursework, with the intent of supporting secondary teacher candidates' mathematical reasoning and attentiveness to students' ideas (Carney, Cavey, & Hughes, 2017). Each online module features a distinct mathematical task with the potential to (a) highlight components of quantitative reasoning, and (b) elicit a range of observable secondary student performance. Appropriate curation and sequencing of secondary student evidence for inclusion in the online modules requires researcher knowledge and application of secondary students' reasoning. In this poster, we describe how we analyzed secondary students' performances related to a quantitative reasoning task, both as preparation for and as a precursor to creating and analyzing candidate work with the VCAST learning modules.

Methods

The 23 participants were mathematics students (grade 7 or 8) from a rural district located in the western United States. Our intent was to gather enough data to represent a range of responses to the task, as doing so is necessary both to build our understanding of student thinking and to inform our development of the online learning module. Each volunteered, with parental permission, to participate in an interview conducted by members of the research team. The interviews consisted of each student completing three mathematical tasks while being video recorded.

The bus stop task asks students to think about the relationally specified quantities of height and age for a group of seven individuals presented pictorially and then to match each person to a point plotted in a Cartesian graph (See Figure 1). This task was intended to elicit students' ability to work with a collection of ordered pairs as a mathematical model of a relation between two quantities, and to interpret and make connections between multiple representations of the data associated with the situation being modeled (Stephens, Ellis, Blanton, & Brizuela, 2017).

Who is represented by each point on the scatterplot?

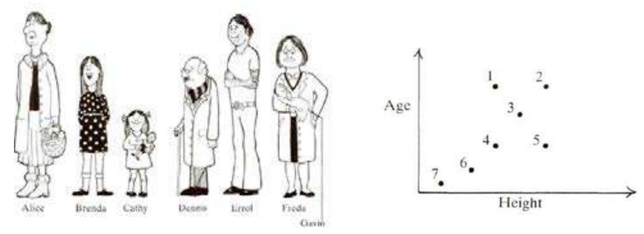

Figure 1. The Bus Stop Task (Swan, M. 1985).

Analysis of the bus stop task data occurred in four distinct phases and involved open coding informed by the literature on quantitative reasoning and grounded theory (Strauss & Corbin, 1994). Phases 1 through 3 involved applying and revising codes. In phase 4, following full application of the coding scheme, we examined frequencies within and across categories and trends in responses. This led to the development of a student reasoning framework for tasks involving graphing relationally specified quantities, along with general descriptors and labels for each primary code. Then we ordered the reasoning descriptors in a hypothesized hierarchy based on observed trends seen in our analysis. For example, we hypothesized that the ability to compare points with a common horizontal or vertical coordinate was more sophisticated than comparing points positioned along a diagonal within the coordinate plane.

Results

The results of our analysis yielded a framework for the levels of sophistication in students' learning performances with the bus stop task. Examples from the data will be provided in the poster to illustrate observable performances for each aspect of inferred student reasoning. This framework will be used as a starting place to analyze candidates' own mathematical work, to describe aspects of student reasoning candidates attend to during online module engagement, and to provide the basis for a key element in the articulation of a learning performance trajectory for candidates' attentiveness.

Table 1: Student Reasoning Framework

Label	Descriptor	Observable Student Performances
Relational Ordering	Ordering a relationally-specified quantity	Compares and orders relative heights and/or ages that do not have assigned numerical values.
1-D Graphing	Observing one-dimensional graphing conventions	Exhibits knowledge that moving up (or right) corresponds to increasing values, while moving down (or left) corresponds to decreasing values.
1-D Comparing	Comparing values for a single quantity graphically	Articulates that points to the right (or above) represent greater heights (or greater ages).
Coupling	Coupling two quantities graphically	References both age and height when determining which person is represented by a given point.
2-D Diagonal Comparing	Comparing values of coupled quantities graphically	Coordinates relative horizontal and vertical position of two points to compare both height and age for points diagonally positioned in the plane.
2-D Aligned Comparing	Comparing values for vertically (or horizontally) aligned points	Exhibits knowledge that points with the same vertical (or horizontal) coordinate represent people who are the same age (or height).

Conclusion

We posit that to improve secondary mathematics teacher candidates' ability to recognize, describe, and analyze secondary student work requires examination of how secondary students reason within mathematical domains. Our explicit articulation of a student reasoning framework informs the research related to these topics, the instructional module design for our project, and the future testing and refinement of our hypotheses about candidates' learning. With respect to module design, we see the importance of contrasting the relative ease with which students can compare points diagonally positioned in the plane with the challenge of comparing values for vertically (or horizontally) aligned points. Thus, the outcomes from the analysis not only contribute to the next phases of our work but also potentially inform the work of others in mathematics education.

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