The Santa Trap: When Scaffolding is Not Enough to Challenge Teachers' Pervasive Beliefs

Chandra Hawley Orrill, University of Massachusetts Dartmouth, corrill@umassd.edu
Rachael Eriksen Brown, Pennsylvania State University-Abington, reb37@psu.edu

Abstract: In this poster, we examine how middle school teachers performed on a task adapted from a study with high school students. We asked 32 middle grades teachers to complete a think-aloud protocol that included the task with three scaffolds to determine whether teachers performed differently than students.

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Major issue addressed and potential significance

Research has shown that students often use linear reasoning in nonlinear situations. In one such study, De Bock, et al. (2002) examined students' reasoning about the Santa Task, which provides two images of Santa: one is small and the other is similar but dilated to be three times taller. The task asks how much paint will be needed to paint the taller Santa, which is 168 cm tall, if 6 ml was needed to paint the smaller Santa, which is 56 cm tall. In their study, De Bock et al. posed the Santa Task with scaffolds to help students (ages 12-13 and 15-16) attend to the area relationship. The researchers found that two students were initially able to answer the Santa task correctly. Even with the five scaffolds, only 32 of the 40 participants were able to determine a correct answer.

Because we are interested in teachers' knowledge of the mathematics they teach, we wondered how teachers would respond to a version of this task. We modified the task scaffolds to be teacher appropriate. Scaffold 1 presented teachers with two different answers from the students in the class: 18 ml and 54 ml and asked the teacher how the students with the wrong answer might be reasoning about the task. In Scaffold 2, we provided the Santa images with rectangular boxes drawn around each. We asked teachers whether this would be a productive approach. In Scaffold 3, the hypothetical student simplified the task by treating the original quantity of paint used as 1 tube and used that to find the new quantity. Again, we asked whether this would be useful for students. In this poster, we examine the teachers' engagement with this set of tasks and its implications for teachers.

Despite the importance of proportional reasoning in the curriculum, little is known about how teachers reason about proportions (Lamon, 2007). There is a growing body of research focused on teachers' understanding of proportions, covariation (e.g., Thompson et al., 2017), and the relationship between teachers' understanding of proportions as it relates to their teaching (Copur-Gencturk, 2015). We add to this literature by examining how teachers understand one relationship with a goal of improving teacher development.

Theoretical and methodological approaches

We work from the knowledge in pieces perspective (diSessa 2006), which posits that understandings develop as fine-grained knowledge resources that are refined and connected through our experiences. From this view, developing expertise requires creating connections between knowledge resources as well as creating more knowledge resources. Thus, learning needs to be comprised of both knowledge resource development and knowledge resource linking opportunities.

To study teachers' understanding of whether proportional reasoning was appropriate for this task, we included the task in a think-aloud protocol sent to 32 teachers from four states. Their responses were captured using Livescribe pens that allowed us to capture both their voices and their written inscriptions. Data were transcribed verbatim. For this paper, we considered whether a participant's response was correct, after which scaffold each participant's reasoning became correct, and what resources were used in solving the task.

Major findings

Similar to the original study of high school students, teachers in this study were misled by the task. Of the 32 participants, nine (28%) (correctly) used an area-interpretation from the outset. Three teachers found their error in scaffold 1 and corrected their responses, none corrected their responses in scaffold 2, and one corrected their responses in scaffold 3. By the end of the task, only 13 (40%) of the 32 teachers solved the task using area interpretation rather than a height-only interpretation.

Teachers' abilities to correctly reason about the Santa situation was critical to their ability to make sense of the student work presented. Teachers who were reasoning mathematically correctly had a much higher likelihood of making sense of the students' responses than those who were not using correct reasoning. For example, in Scaffold 1, all 13 teachers who used area reasoning were able to identify the mistake the students had made. In contrast, none of the teachers using height-only reasoning did this. In Scaffold 2, 12 of the 13 area-reasoning teachers believed that drawing the frame around the Santa would be useful. Many noted that it would help students see both height and width. Of the height-only reasoners, six (32%) recognized that the frames helped to show area, but this did not change their thinking. Four of the 19 height-only teachers (21%) saw the frame as highlighting one dimension (e.g., height), and six rejected (32%) the frames saying the frames inappropriately focused students on area, which they perceived to be incorrect. Scaffold 3 was harder overall. Among teachers (correctly) attending to area, nine (69%) believed that using easier numbers was appropriate and usable for their students. Two were okay with the hypothetical student using the approach, but would not want their own students to use it and three teachers (23%) were unable to make sense of the situation. In contrast, none of the height-only reasoners believed this was a useful approach. Eight of the 19 (42%) pointed out that it yielded incorrect answers and six (32%) could not make sense of what was happening or were unclear. Interestingly, four (21%) of the teachers rejected this approach because they believed that one cannot reason about tubes as the unit in this way.

Conclusions and implications

Consistent with DeBock et al. (2002), participants in this study were lured to interpret the Santa Task as being about one dimension. As mentioned above, four of the 19 height-only reasoners rejected using a simpler task (Scaffold 3). Data suggests that the teachers did not recognize that this was an appropriate approach because 9 : 1 is the same relationship whether it is describing 9 tubes to 1 tube or 9 ml to 1 ml. The teachers' approach to the task itself suggests that teachers are not making appropriate sense of the task. This adds to literature suggesting that teachers need additional opportunities in determining the nature of tasks (e.g., Izsák & Jacobson, 2017). Finally, teachers who approached the task incorrectly were unable to follow student reasoning in the three scaffolds. The main implication of this is that it suggests teachers' abilities to make sense of student thinking is tied to students' abilities to produce expected answers.

References

Copur-Gencturk, Y. (2015). The effects of change in mathematical knowledge on teaching: A longitudinal study of teachers' knowledge and instruction. Journal for Research in Mathematics Education, 46, 280-330.

De Bock, K., Van Dooren, W., Janssens, D., & Verschaffel, L. (2002). Improper use of linear reasoning: An in-depth study of the nature and irresistibility of secondary school students' errors. Educational Studies in Mathematics, 503, 311-334.

diSessa, A. A. (2006). A history of conceptual change research: Threads and fault lines. In R. K. Sawyer (Ed.), The Cambridge handbook of the learning sciences (pp. 265-282). New York: Cambridge University Press.

Izsák, A., & Jacobson, E. (2017). Preservice teachers' reasoning about relationships that are and are not proportional: A knowledge-in-pieces account. Journal for Research in Mathematics Education, 48(3), 300-339.

Lamon, S. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In F. K. Lester (Ed.), Second handbook of research on mathematics teaching and learning (pp.629-667). Reston, VA: National Council of Teachers of Mathematics.

Thompson, P. W., Hatfield, N. J., Yoon, H., Joshua, S., & Byerly, C. (2017). Covariational reasoning among U.S. and South Korean secondary mathematics teachers. Journal of Mathematical Behavior, 48, 95-111. doi: 10.1016/j.mathb.2017.08.001

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