Embodied Design for Mathematical Imagination and Cognition

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Workshop Outcomes

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Embodied Design for Mathematical Imagination and Cognition

A research agenda for investigating how engaging students' bodies can improve mathematics learning.

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Abstract

Embodied Mathematical Imagination and Cognition is an emerging theoretical framework that seeks to explain the role of people's physical being and behavior in how they learn and teach, and how they express what they have learned during communication and assessment. Research-oriented frameworks for incorporating embodied cognition into the design of learning environments can promote mathematics learning, a core scholastic content area often regarded as particularly abstract and disembodied. Technology in these environments can provide developers and teachers with rich, real-time data from users to track engagement, support formative assessment, and target both conscious and unconscious learning processes. Interdisciplinary scholars and practitioners from education research, cognitive science, the learning sciences, developmental psychology, dance, movement science, computer science, and mathematics and science education attended a workshop. They articulated consensus statements, noted tensions, and developed recommendations for a research agenda.

Keywords

Embodied cognition, mathematics, design-based research

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Introduction

Embodied cognition is an emerging theoretical framework that seeks to explain the role of people's physical being and behavior in how they learn and teach, and how they express what they have learned during communication and assessment. Reviewing the state of mathematics education, Schoenfeld (2016) notes that "both embodied cognition (e.g., Nemirovsky & Ferrara, 2009) and embodied design (e.g., Abrahamson, 2009; Alibali & Nathan, 2012) are receiving increased attention in mathematics education" (p. 514). A number of existing systems used in classrooms, afterschool settings, and randomized control experiments have been built to integrate kinesthetic motion capture, real-time optical motion tracking, and touch screens that use perceptual-motor simulation of continuous motion to enact symbol manipulation and abstract reasoning. Although approaches differ, researchers today are (a) designing mathematics activities where students develop mathematical ideas, (b) sharing insights on how sensi-motor activity shapes students' understanding of mathematical operations and relationships, and (c) seeking causal accounts of how bodily movement facilitates mathematics reasoning. Addressing embodied ways of knowing for mathematics is of particular theoretical import because mathematics is a core scholastic content area often regarded as particularly abstract and disembodied.

The overall general goal for our workshop was to form a 10-year research agenda that would provide a coherent set of evidence-based design principles for enhancing mathematics education and broadening participation in all STEM fields.

Mathematics education—the connective representational "language" among the STEM fields—benefits from such innovation as it supports the creative process in STEM for imagining what can be. The Mathematical Association of America promotes creativity and curiosity as a vehicle to mathematical proficiency and communication but remains out of reach for many students. This workshop focused on the potential of designing for Embodied Mathematical Imagination and Cognition (EMIC), with the goal of specifying principles that broaden the mathematical participation of young learners through body-based forms of reasoning and expression.

Several broad areas are ripe for immediate applicability to design for STEM learning. Cognitive and attentional processes are quite limited, so ways instructional design provides

resources to offload cognitive and attention processing to the body and other external resources offer immediate promise for improving intellectual performance.

There is also evidence that conceptual development from novice learners of all ages benefits from early exposure to concrete learning experiences before demonstrating generalized mastery and transfer—what our participants referred to as "experience first, signify later" (Abrahamson, 2009). This favors approaches such as concreteness fading, and progressive formalization, and is a rejection of formalism-first approaches to curriculum design and instruction.

Workshop attendees

A workshop on May 20-22, 2019 at the University of Wisconsin–Madison brought together leading scholars in mathematical reasoning, teaching, and learning who work on embodied design (see the Appendices for a list of workshop organizers and the schedule of presentations and activities). We sought to attract an interdisciplinary set of scholars and practitioners from education research, cognitive science, the learning sciences, developmental psychology, dance, movement science, computer science, and mathematics and science education. The research presented spanned K–16 topics in content areas such as arithmetic and algebra, proportional reasoning and fractions, geometry, complex numbers and functions, statistics, and calculus.

The main goal of our workshop was to form a 10-year research agenda; this agenda would provide a coherent set of evidence-based design principles for enhancing mathematics education and broadening participation in all STEM fields. The findings, principles, and proposed future research extend to studies of mathematical intuition and reasoning, learning in and outside of formal educational settings, teacher professional development, classroom instruction, and assessment. Forty-nine applications were accepted and nearly all of them were fully funded to attend.
Our attendees were primarily from the United States and we had six international participants. We accepted five K-20 teachers, each of whom conducted their own educator roundtable to discuss how EMIC informs teaching, and how teaching, in turn, informs EMIC. We accepted 19 graduate students and postdoctoral/early career faculty, who each presented at the opening day poster session. We accepted 21 senior faculty who each presented for 10 to 15 minutes. Lastly, a mixture of students, postdoctoral scholars, and faculty (the majority of whom also presented a poster or a talk) facilitated one of 9 embodied mathematical activities throughout the workshop to ground our discussions.

Workshop structure

In the workshop, we focused on the design of systems for classroom settings, with attention to equity and access for underrepresented groups, while examining evidence of learning both in and outside of school. Each applicant was asked to present a poster or paper, or facilitate an educator round table or embodied mathematical activity, and we designed a single-track program based upon their contributions. One attendee noted, "Because everyone participated in some form, the workshop had a greater sense of collaboration and investment than other experiences I've had…."

Our two keynote speakers were Dr. Brian Bottge, who specializes in mathematics and special education, and Dr. Maxine McKinney de Royston, who specializes in mathematics and multicultural education related to race, identity, and equity. They were chosen specifically to help push our EMIC conversations towards the importance of equity in mathematical learning. Dr. Bottge's presentation focused on tasks that have motivated students to develop a deep understanding of math concepts and Dr. McKinney de Royston's presentation explored emerging perspectives in the learning sciences that seek to expand the epistemological and ontological premises of human learning and teaching. A survey respondent noted that they "particularly liked...the choice of keynotes (providing fresh perspectives on embodied learning)." Our two discussants were Dr. Arthur Glenberg, an expert in language and embodied cognition, and Dr. Jim Slotta, an expert in technology and learning. They infused our workshop with considerations from relevant, adjacent fields of inquiry.

Because everyone participated in some form, the workshop had a greater sense of collaboration and investment....

Our EMIC workshop (and EMIC research more broadly) was characterized by lively discussions that reflect the many diverse and complementary methodological considerations of the emerging inter-discipline—not surprising, given the plethora of fields represented, as described above. Second, we found a distinct desire for a theoretical convergence that leverages interdisciplinary connections into a coherent collective voice. In particular, our attendees seek to identify consistent learning principles that inform EMIC while still preserving the distinct disciplinary identities.

Key issues

Specific activities at the workshop identified consensus statements, design approaches, and surprises or tensions. The assembled experts sought common ground among the learning sciences, psychology, dance, movement science, linguistics, computer science, education and special education, and mathematics.

Consensus statements

Workshop participants generated statements during the workshop. We report statements that had at least 50% agreement on a post-workshop survey (n=33).

- More experimental evidence is needed to pinpoint the specific interactions between students, subject matter, and situated, grounded, and embodied curricular design (71% agreement)
- I operate under the assumption that all cognition is inherently embodied (68% agreement)
- I am still learning about cognition, and how we learn (61% agreement)

Design approaches

Participants agreed they used the following in their work:

- Gesture theory (75%)
- Activities should be designed to leverage our naturally occurring perceptions toward conceptual understanding (72%)
- The tenet that social emotion can impact conceptual integration (63%)
- Principles on designing dynamic gesture-based interfaces for mathematics (56%)
- 'Experience first, signify later'(56%)
- Encouraging physical and spatial exploration of the structure of algebra (50%)
- The belief that dynamic gestures allow participants to physically experience generalized properties through enactment (56%)
- Distributed cognition (50%)

Summary of findings and recommendations

- **Overarching finding.** Any research or policy agenda should consider how Embodied Mathematical Imagination and Cognition (EMIC) broadens participation in mathematics education.
- **Immediate Recommendations.** EMIC learning research should employ participatory design methods, specifically including teachers and learners. Gesture use for instruction, learning interventions, and formative assessment appears to be an especially promising way to support grounded and embodied learning and teaching.
- **Near Term Recommendations.** Practitioner-researcher partnerships are important for design, implementation, and evaluation of EMIC-based interventions.
- **Long-term Recommendations.** Embodied design of educational experiences has the potential to radically transform the nature of mathematics learning, teaching, and assessment, but will need focused and sustained funding for such efforts to mature.

Tensions and unanswered questions

An emergent tension that became a critical theme throughout the workshop was the practical educational implications of EMIC, and how to support the innovations and findings from the field in transitioning into direct impact on our educational system. Attendees agreed that:

- Teachers are professionals that should be deeply involved in developing, adopting, and adapting embodied learning activities (81%)
- Teachers should be able to adopt and adapt body-based learning activities (75%)
- What counts as math (and who decides) is a constant tension in this interdisciplinary field (63%)

Attendees also noted unanswered questions:

- Regarding different types of embodiment that need investigation (84%)
- How best to work with practitioners (75%)
- Emerging research practices and methods (59%)
- Research practices that need investigation (59%)

Recommendations for future work

Our recommendations span grain sizes from individuals (a teacher, a researcher) all the way up to the policy-makers in our educational system, and the funding agencies responsible for determining which research is important. Broadly, we recommend that any research or policy agenda should consider how the EMIC perspective broadens participation for those who are minoritized or differently abled; increases the awareness of gesture as signifying content-related communication; considers the classroom environment, curricular objects, and teacher needs when introducing technology; supports learning by beginning from intuition; and enables movement. Particularly, there is a distinct sense of urgency for participatory design and an emphasis on the need to keep including teachers when designing experiences for the classroom. In addition, there is a push for additional evidence that gesture directly facilitates learning and instruction. Attendees noted that the EMIC collective needs further reflection, planning, and resources, and that we need to strategize for community development, intellectual cohesion, identity, longevity, and impact. Detailed recommendations follow; each recommendation set represents a component of our overall goal to form a ten-year research agenda that would provide a coherent set of evidence-based design principles for enhancing mathematics education and broadening participation in all STEM fields.

Our goal: a 10-year research agenda to build a coherent set of evidence-based design principles for enhancing mathematics education and broadening participation in all STEM fields.

Immediate Recommendations. EMIC learning research should employ participatory design methods, specifically including teachers and learners who can offer feedback and advice throughout the design process. Doing so is expected to increase the fidelity of implementation and overall usefulness of the product or experience in both classrooms and informal learning environments. Consensus guidelines for

implementing and facilitating such environments would be highly valuable.

Near Term Recommendations. First, the research would benefit by cultivating partnerships with policy-makers, as well as practitioners, so as to prepare for eventual influence on how and what students learn in mathematics classrooms. Second, the field needs a breadth of ways to share and curate empirical inquiry into the conditions for designing EMIC for instruction, assessment, and learning environments. Third, the field needs to develop and refine data collection tools and research methods for studying body-based processes, body states, and cognitive states. Fourth, and potentially most important, the field must investigate and report on how EMIC addresses equity in the classroom.

Long-term Recommendations. This field has the potential to reshape or radically change what we know about how people learn, but doing so will require refining hypotheses and replicating findings, to build a compelling, cumulative body of work.

The field also may deeply inform understanding of how unconscious processes interact with consciousness and metacognitive processes.
This field could also reshape what is known about learning in spatial cognition. An embodied framework for mathematical reasoning is likely to expand those systems to include touch and movement senses in ways that are greatly expansive and inclusive for describing how we experience the world.

The bottom line is that this field is very promising yet has been operating on the margins of existing funding programs. In order for EMIC to have a direct and productive impact on instruction, assessment, and our understanding of the mechanisms of learning, we must have a stable source of funding that begins at exploration and ends at scaling up.

For further information about this workshop, our discussion, and citations for statistics and quotes, please read the full white paper.

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Resources

- Our website: www.embodiedmathematics.com (Sign up for announcements to receive information about future workshops or virtual panels.)

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Appendix 1

The Organizing Committee

Mitchell J. Nathan (Chair), University of Wisconsin-Madison
Dor Abrahamson, University of California, Berkeley
Martha W. Alibali, University of Wisconsin-Madison
David Landy, Indiana University
Erin Ottmar, Worcester Polytechnic Institute
Hortensia Soto, University of Northern Colorado
Candace Walkington, Southern Methodist University
Caro Williams-Pierce, University at Albany, SUNY (soon to be University of Maryland, College park)

Appendix 2

Schedule of Presentations and Activities

The workshop was held May 20-22, 2019 at the University of Wisconsin–Madison. All attendees presented.

The primary location for this workshop is UW's Union South, in two rooms (Northwoods and Industry) on the Third Floor. See Venues for more information about the location, the individual

rooms, and assorted other useful details. And don't forget to go to the Home Page to join our WhatsApp group!

Text throughout the schedule that looks like this is a hyperlink - *you can always click on them for more information about that specific event!*

Day 1: May 20 (Monday)

1:00 PM: Welcome & Workshop Objectives (Industry Room)

- Mitchell J. Nathan
- Michael Swart & Greg Zorko
- And the Organizing Committee: Martha W. Alibali, Dor Abrahamson, David Landy, Erin Ottmar, Hortensia Soto, Candace Walkington, Caro Williams-Pierce

1:30 PM: Talk Session #1 (Theme: Big Ideas) (Industry Room)

- Laurie Edwards - Embodiment & Constructivism: A Meta-Theoretical Critique
- Dor Abrahamson - Moving Perception Forward in Learning Sciences Discourse
- Ilona Iłowiecka-Tańska - Learning in The Science Center: Interaction with Hands-On Exhibits to Build Mathematical Concepts
- Karl Schaffer - A Very Brief Introduction to an Equal Partnership Between Mathematics and Dance
- Mitchell Nathan - Embodied Meaning & Abstraction in Mathematics: Looming Questions

3:00 PM: Junior Scholars Poster Session *& Afternoon Reception* (Northwoods Room)

5:00 PM: Dr. Brian Bottge Keynote Presentation (Industry Room)

Dr. Brian Bottge is the William T. Bryan Endowed Chair and Professor in Special Education in the Department of Early Childhood, Special Education and Rehabilitation Counseling, University of Kentucky, and emeritus professor of Rehabilitation Psychology and Special Education at the University of Wisconsin-Madison.

Dr. Bottge is best known for Enhanced Anchored Instruction (EAI), which is a strategy for teaching mathematics to low-performing adolescents. EAI provides rich and engaging contexts (i.e., computer and hands-on applications) where students develop their computation and problem-solving skills. Dr. Bottge is currently the PI on an IES Goal 5 grant, *Developing Enhanced Assessment Tools for Capturing Students' Procedural Skills and Conceptual Understanding in Math.*

Building Students' Conceptual and Procedural Knowledge in Engaging Learning Environments. A series of studies over the past 20 years has shown how teachers can uncover and improve the problem-solving skills of their students, including those who are low achieving. The presentation will focus on the tasks that have motivated students to develop a deep understanding of math concepts.

7:00 PM: Working Dinner at the downtown Great Dane

Day 2: May 21 (Tuesday)

8:00 AM: Mathematics Activity and Breakfast

- Roni Zohar (Northwoods Room) - Dance/Science

9:00 AM: Talk Session #2 (Theme: Culture, Math for All, Inclusion) *with* Discussant Art Glenberg (Industry Room)

- **Justin Dimmel - The Geometry of Movement: Designing a Gesture-Based Virtual Environment for Making and Transforming Spatial Inscriptions [schedule change]
- Paul Ginns - Getting the Point: Pointing and Tracing Enhance Learning of Mathematics
- Hortensia Soto - Body Transformations
- Roni Zohar - Embodied Learning of Physics Concepts
- Yanghee Kim - Young Children's Embodied Interactions with a Humanoid Robot

11:30 AM: Dr. Maxine McKinney de Royston Keynote Presentation *& Lunch* (Industry Room)

Dr. Maxine McKinney de Royston is Assistant Professor of Curriculum and Instruction (Mathematics Education and multicultural education), University of Wisconsin-Madison

Dr. McKinney de Royston's research is concerned with understanding how to create productive learning environments for minoritized students, particularly Black students. Focusing on math and science classrooms, she studies the pedagogical and relational characteristics of learning environments as they relate to larger discourses about race, identity, and learning. At present, Dr. McKinney de Royston's research centers around two interrelated strands: the sociopolitical nature of teaching and learning and how learning spaces, such as mathematics classrooms, are inherently racialized.

"I wish I was white": Political and Ethical Considerations for (Re)Conceptualizing Mathematical Knowing and Doing. Policies and research in mathematics education continue to forward a rhetoric of "mathematics for all", yet the underlying theories of learning that drive k-12 mathematics remain narrow, racialized constructions of mathematical knowing and doing. This talk will focus on emerging perspectives in the learning sciences that seek to expand the epistemological and ontological premises of human learning and teaching that engage the "how," "for whom," and "towards what ends" of mathematical knowing and doing.

1:00 PM: Mathematics Activity and Coffee Service

- Karl Schaffer (Northwoods Room) - Dance/Math. *Wear comfortable, loose clothing!*
- Justin Dimmel (GEAR Learning at the Educational Sciences Building - see Venues) - a VR environment called Handwaver

2:00 PM: Talk Session #3 (Theme: Technology and Learning Environments) *with* Discussant Jim Slotta (Industry Room)

- Caro Williams-Pierce - Designing for - and Seeing - Mathematical Play
- Erin Ottmar - Graspable Math: Integrating Perceptual Learning, Gesture, and Action within Algebra Problem Solving
- Alik Palatnik - 3D Sketching Approach to Solid Geometry Learning
- Tyler Marghetis - Doing Math as Design: How Math-Doers Create Their Own Ecosystems for Thinking

· Ivon Arroyo - WearableLearning.org: Learning Technologies for Embodied Math Learning

5:00 PM: Mathematics Activity and Coffee Service

· Leah Rosenbaum (Northwoods Room) - Geometris

· Rachel Chen and Sofia Tancredi (Industry Room) - Mathematics Imagery Trainer

6:30 PM: Break

7:00 PM: Working Dinner at Steenbocks on Orchard

Day 3: May 22 (Wednesday)

8:00 AM: Mathematics Activity and Breakfast

· Hortensia Soto (Northwoods Room) - Learning by Sliding, Turning, and Flipping

· Oh Hoon Kwon (Industry Room) - Chasing Angles for Proof

9:00 AM: Educator-Led Roundtables on Translational Goals (Industry Room)

· Jennifer St. John

· Kelsey Schenck

· Michael Bowling

· Elizabeth Dutton

· Oh Hoon Kwon

10:00 AM: Break

10:30 AM: Talk Session #4 (Theme: Instruction and Gesture) *with* Facilitator Martha W. Alibali (Industry Room)

· Janet Walkoe - Teacher Noticing of Student Thinking as Expressed Through Gesture and Action

· Martha W. Alibali - Understanding the Role of Teachers' Gestures in Students' Learning: Lessons from a Teacher Avatar

· Susan Wagner Cook - Individual Differences in Learning with Hand Gesture: The Role of Visual-Spatial Working Memory

· Nicole Engelke Infante - Communicating Calculus: Two Sides of the Equation

· Caroline Yoon - Metaphor Tangles in an Inverse Problem Calculus Task

· **Candace Walkington - Collaborative Gestures in a Motion-Capture Geometry Video Game [schedule change]

12:30 PM: Grab a lunch from a local restaurant (see Venues) or the fabulous lunch carts on the Library Mall

1:00 PM: Working Lunch - Synthesis: Working Out a 10-Year Research Agenda (Industry Room)

· Facilitator: Mitchell J. Nathan

3:00 PM: Break

3:30 PM: White Paper and Book Project (Industry Room)

· Facilitators: Mitchell J. Nathan and Caro Williams-Pierce

5:30 PM: Mathematics Activity and Coffee Service

· Dor Abrahamson (Northwoods Room) - Embodied Icosahedra

· Candace Walkington (Industry Room) - The Hidden Village

7:00 PM: Working Dinner at the classic and beloved UW Memorial Union Terrace!

· Back-up location in inclement weather is Der Rathskeller (inside the Memorial Union)