Abstract: One factor impacting the effectiveness of online teacher professional development is that teachers primarily share and compare ideas in their discussions. This study aimed to better understand the process by which a knowledge-building community of practice (CoP) emerges in an online mathematics teacher professional development course. We characterized the generativity of teachers’ interactions and employed social network analysis to examine evolution in the pattern of these interactions. We found signs of a knowledge-building CoP, evidenced by a stabilizing and evolving core-periphery structure and consistent increase in the extent to which participants engaged generative interactions to build knowledge with one another. This work has implications for developing technologies that can assist teacher educators in cultivating online knowledge-building CoPs.

Major issues
Teachers participating in community-based professional development is associated with increases in student achievement (Ronfeldt, et al, 2015). However, not all professional development has the intended effect on teacher learning and instructional change (Kennedy, 2016). One key factor impacting the productivity of online professional development is that in both face-to-face and online teacher professional development (OTPD) spaces, research indicates that teachers are sharing and comparing information and are not frequently engaging in generative interactions (Lantz-Andersson, et al., 2018) that are so important for knowledge building (Scardamalia & Bereiter, 2014). The purpose of this study was to characterize the emergence of a knowledge-building community of practice (CoP) by examining evolution in the patterns in mathematics teachers’ generative interactional practices in OTPD. This work is significant because it can contribute to designing online learning environments that use the affordances of virtual learning to support the emergence of knowledge-building CoPs, with potential implications for impacting the quality and equity of mathematics teachers’ instruction.

Knowledge-Building CoPs and social networks
Underlying our approach is a view of learning as increased participation in a knowledge-building CoP (Lave & Wenger, 1991; Scardamalia & Bereiter, 2014). A CoP is often defined as a group of individuals with shared goals, shared practices, and a shared set of tools (e.g. concrete and conceptual) (Wenger, 1998). CoPs provide context for teachers participating in prolonged and collaborative professional development that is linked to teachers improving their instruction (Desimone, 2009). To be productive for teacher learning and instructional change, we argue that one defining feature of a CoP is teachers engaging in generative interactional practices. Therefore, we use the term knowledge-building CoP to refer to a CoP that is defined, in part, by interactional practices that are productive for social knowledge construction. This term is useful because it allows us to specify the type of community we are interested in understanding and supporting.

We parse interactional practices into two broad categories, those that share and compare and those that are generative. Sharing and comparing is consistent with following norms for agreement that are pervasive in teacher collaboration (Grossman, et al., 2001). Generative interactions include those that co-construct knowledge by disagreeing, critiquing alternative perspectives, negotiating and building on others’ ideas, and synthesizing and applying ideas to new contexts (Gunawardena, et al., 1997; Lotman, 1988). While we agree that sharing ideas is necessary for generative collaboration (Zhang et al., 2017), we argue that knowledge-building CoPs include more consistent engagement in generative interactional practices.

There are connections between the structure of a social network and the presence of a CoP (Jan, & Vlachopoulos, 2018). Two indicators of a CoP include a cohesively connected network - “attributes of the whole network, indicating the presence of strong socializing relationships” (Haythornthwaite, 1996, p. 332) - and the presence of a core-periphery structure (Jan, & Vlachopoulos, 2018; Lave & Wenger, 1991). The core of a CoP is highly connected and consists of “old-timers” who are experts in the practices that define the community, while the periphery is sparsely connected and interacts with the core on occasion to offer new perspectives that shape and reshape a CoP. Framing our study in this way allowed us to identify patterns in the process by which
participants’ network of generative interactions evolves and interpret the extent to which the structure resembles a community.

**Designed learning environments**

Research indicates that the design of a learning environment can have an impact on the generativity of a collective’s interactions and the structure of their social network (Yücel & Usluel, 2016). Chieu, Herbst, and Weiss (2011) outlined a set of design principles for supporting productive collaboration in OTPD that includes embedding rich-media artifacts into the online context, creating opportunities for teachers to engage with these artifacts, and including moderators who scaffold the focus of teachers’ collaboration. Likewise, Clay, Silverman, and Fischer (2012) introduced a model for structuring online asynchronous learning that includes an initial period for private mathematics work, discussing and critiquing one another’s work, and revision. The OTPD course in this study was designed through the lens of these principles and included this model for collaboration.

**Methodology**

This study was guided by the following research question: How does the cohesiveness and core/periphery structure of participants’ generative interactions evolve and begin to resemble a knowledge-building CoP? We examined 21 mathematics teachers and the course instructor participating in a 10-week asynchronous OTPD course focused on the concept of function ($n=22$). The instructor frequently engaged in generative interactions with participants and, therefore, modeled such interactional practices for participants. Following Chieu et al. (2011), the course embedded interactive applets into the mathematics tasks to scaffold participants in reasoning about functions. To apply the online model for collaboration, participants posted an initial response to the mathematics task halfway through a week, then provided at least two comments on their peers’ work.

**Data and data analysis procedures**

Data included participants’ conversations on the course discussion board (DB) that were in response to each initial post (521 posts total). Data analysis included qualitative coding (Maxwell, 2005) and Social Network Analysis (SNA) (Scott & Carrington, 2011). The initial stage of analysis involved the first two authors collaboratively coding (in Nvivo 12) the data set using the Interaction Analysis Model (IAM) (Gunawardena, et al., 1997), which characterizes interactions according to how they contribute to social knowledge-construction (Table 1 has phase descriptions and examples from our data). Phase I of the IAM aligns with our conceptualization of sharing and comparing. Phases II-V align with our conception of generative interactions. For the second stage of analysis, we extracted the coded data, prepared cumulative datasets (e.g. the week two data set included generative interactions from week one and week two) for the eight of the ten weeks that included mathematical activity, and entered the data into UCINET 6.0 for the SNA on participants’ generative interactions (interactions coded at Phases II-V). To assess network cohesiveness, we employed the density procedure, which calculates the proportion of ties present to all possible connections in the network. We also conducted a core-periphery analysis (Borgatti & Everett, 2000), which bifurcates a social network into core and peripheral subgroups. One output of this analysis is membership data, which identifies members of the core and periphery. We organized the results from the SNA procedures into spreadsheets, developed sociograms, and generated alternative forms of the data to identify patterns in the evolution of generative interactions.

**Table 1: The IAM and examples from our data**

<table>
<thead>
<tr>
<th>IAM Phases</th>
<th>Example from our data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(PI) Sharing and comparing</td>
<td>Yes! I think equidistant is the word! I remember hearing it in geometry and topology classes. I wish I could have recalled it...Well, now I know! Thank you!</td>
</tr>
<tr>
<td>(PII) Discovery and exploration of dissonance</td>
<td>One thing that caught my attention was your use of “minimum” and “maximum.” I know we are restricted between the two exits, but what if we went past the exits, would the distances still be maximums and minimums?</td>
</tr>
<tr>
<td>(PIII) Negotiating meaning</td>
<td>I noticed you wrote the value of $y$ will never exceed 1 or -1 because of the nature of the sine function. I wondered if you could explain what you mean by the &quot;nature of the sine function.&quot; My thought was that the relationship between $x$ and $y$ explains this.</td>
</tr>
<tr>
<td>(PIV) Testing and modifying ideas</td>
<td>I wondered about your last statement, &quot;the side of the road on which the city lies does not matter...&quot; This statement made me think and I had to open the applet to test your statement... I was thinking that you meant you could take city A and put it</td>
</tr>
</tbody>
</table>
where city B was, which flips the graph, but you meant that you could put the cities in the same position on opposite sides of the road and get the same graph.

(PV) Metacognitive statements and applying new ideas
I was wondering if the sides of the roads made the graph tilt certain ways. However, I just read Henry's response down below and the tilt seems to be caused by the distances. So, I guess the sides really do not matter!

Findings
The cumulative counts from our coding are shown in Table 2. Given what has been reported in extant research (Lantz-Andersson, et al., 2018), it is expected that there would be more interactions that share and compare. However, our results show a consistent increase in the generative interactions that occurred in the OTPD course.

We also analyzed participants’ social network of generative interactions (interactions coded as PhII-V). Table 2 shows how UCINET parsed participants into the core and periphery, where a C indicates the participant was in the core for that particular week of the course (the 9 participants not shown were never a part of the core during the course). We characterized the evolution of participants’ generative interactions in three stages. Stage 1: Low-density network and varying core/periphery (e.g. two members in the core in week one, three new members in week two, and one new member and three “leavers” in week three). Stage 2: Low-density and stabilizing core/periphery (e.g. three members define the core in week five and six and stay in the core for the rest of the course). Stage 3: Moderate density and evolving core (e.g. the core increases to seven members in week eight and nine).

Table 2: Core/periphery structure, density, and phase counts

<table>
<thead>
<tr>
<th>Stage</th>
<th>Week</th>
<th>Membership with the core</th>
<th>Density</th>
<th>Cumulative Phase Counts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 3</td>
<td>Wk 9</td>
<td>C C C C C C</td>
<td>22.1%</td>
<td>PI:355; PII-V:166</td>
</tr>
<tr>
<td></td>
<td>Wk 8</td>
<td>C C C C C C</td>
<td>20.1%</td>
<td>PI:320; PII-V:155</td>
</tr>
<tr>
<td>Stage 2</td>
<td>Wk 6</td>
<td>C C C</td>
<td>16%</td>
<td>PI:258; PII-V:118</td>
</tr>
<tr>
<td></td>
<td>Wk 5</td>
<td>C C C</td>
<td>13.4%</td>
<td>PI:219; PII-V:98</td>
</tr>
<tr>
<td>Stage 1</td>
<td>Wk 4</td>
<td>C C C C C C C</td>
<td>11.9%</td>
<td>PI:200; PII-V 86</td>
</tr>
<tr>
<td></td>
<td>Wk 3</td>
<td>C C C C C</td>
<td>9.3%</td>
<td>PI:172; PII-V:69</td>
</tr>
<tr>
<td></td>
<td>Wk 2</td>
<td>C C C C C</td>
<td>5.3%</td>
<td>PI:133; PII-V:46</td>
</tr>
<tr>
<td></td>
<td>Wk 1</td>
<td>C C</td>
<td>3.6%</td>
<td>PI:80; PII-V: 36</td>
</tr>
</tbody>
</table>

There is evidence that one contributing factor to this emerging network structure is P8, the course instructor. During stage one, he modeled generative responses and contributed to establishing a core/periphery structure; after that, he started to phase himself out. At week 4, we have a large core that includes the instructor. After that, he moves into the periphery, while we see the network continue to stabilize and evolve as the network coalesces into seven core members who continue to engage in generative discourse. This provides an example of the impact of the scaffolding principle described by Chieu et al. (2011) on the network of generative interactions, and suggests that the scaffold can be removed and the generative practices will continue.

Potential significance and implications
This study defined three stages in the evolution of the cohesiveness and core/periphery structure of mathematics teachers’ social network of generative interactions in an OTPD course. We found that the density of participants’ generative network increased during each stage, indicating the emergence of a more cohesive effort to build knowledge, by, for example, expressing disagreement and negotiating meaning with colleagues. In addition, we noticed a core/periphery structure that was initially unstructured begin to stabilize and evolve, indicating the emergence of a core group of “old-timers” who began more frequently engage generative interactions with one
another. Given that these are features of a CoP (Jan, & Vlachopoulos, 2018; Lave & Wenger, 1991), we argue that this study found signs of an emerging online knowledge-building CoP defined by generative interactional practices. This is significant because it indicates that this OTPD context supported teacher collaboration that moved beyond norms for agreement that are so persistent in OTPD. Moreover, we contribute a characterization of the process by which a knowledge-building CoP in OTPD emerges. Future work will include developing a typology that characterizes how social networks of generative interactions evolve in OTPD and relating these types of networks to design features of learning environments. This work will contribute to our broader goal of developing technological tools that can assist facilitators in supporting the emergence of knowledge-building CoPs by recognizing features of evolving social networks in OTPD and making recommendations for how to intervene in ways that will impact the trajectory of the generativity of collaboration.

References