Teaching the Purpose and Meaning of Algebraic Variables Through Systems-of-Equations Story Problems: Multimedia Approaches

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Abstract: Traditional approaches to teaching variables often begin with problems students can already solve arithmetically. Varying instructional method across conditions, we developed multimedia story sequences with systems-of-equations problems whose solution requires using variables. Sixth graders from two US schools (n = 163) completed two one-hour learning sessions and a delayed posttest weeks later. Across all conditions, sixth graders' performance on the posttest was comparable to samples of 12 eighth graders post-learning and 20 ninth graders pre-review.

Keywords: algebra, multimedia videos, concepts, procedures, systems of equations

Introduction

Understanding the use of variables in algebra is both important in its own right and foundational for higher levels of mathematics, but many students demonstrate a variety of misconceptions about the interpretation of variable and algebra's relation to arithmetic (Knuth et al., 2005; Sfard & Linchevski, 1994). Traditional approaches to teaching variables in sixth grade (11- to 12-year-olds in the US) often begin with formal definitions and one-step symbolic problems (e.g., $x + 2 = 7$) that students can already solve by retrieving related arithmetic facts (e.g., $5 + 2 = 7$). This approach leaves many students puzzled about variables' purpose and meaning. More recent research suggests students better understand abstract notation when it is first 'grounded' with concrete materials and then gradually abstracted (concreteness fading; Fyfe et al., 2014) and progressively formalized (Nathan, 2012).

We posit that an additional, unexamined factor in students' difficulties understanding variables may be that early examples of algebraic problem solving do not demonstrate the purpose of a new representation and concept beyond arithmetic. That is, when learning algebraic representation from one-step problems like $x + 2 = 7$, even when grounded, x may appear to be simply a cue to retrieve $5 + 2 = 7$ or its inverse, $7 – 2 = 5$. Thus, a better way to convey the purpose of variable notation may be with a system of equations in two variables, where the retrieval of arithmetic facts alone would not provide direct answers. Our project set out to determine whether introducing algebra to sixth-grade students using systems-of-equations problems (eighth-grade standards) would improve their understanding of the purpose of variables, and if the outcome of this instruction differed depending on whether the formalisms are taught first or gradually introduced. In other words, our present study tested whether the abstractness of algebraic symbols introduced *before* they become essential to solving problems results in a lack of perceived purpose for variables.

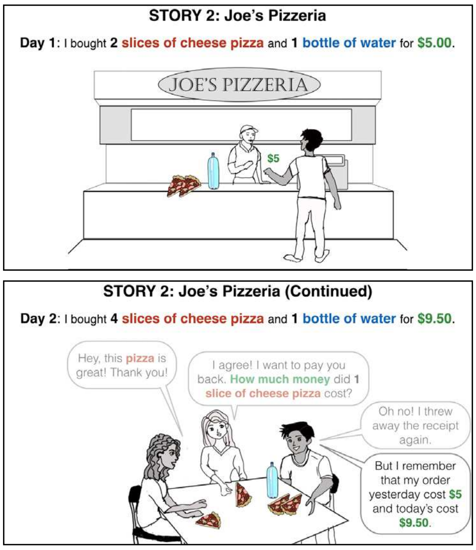

Figure 1. Screenshots of a story problem with purchases of two food items over two days, leading to a system of equations.

Materials

We developed three versions of multimedia videos introducing algebra using systems of equations, two featuring a narrator purchasing food and trying to figure out how much individual items cost (Figure 1).

Our '*Purpose-Driven Progressive Formalization*' (PDPF) approach grounds instruction in problem contexts and gradually abstracts, progressively formalizing the use of literal symbols by first introducing word equations (Figure 2). Students are asked to attempt the problems prior to instruction, then are given 'intuitive' explanations of the procedures (Figure 2) in terms of the concrete details of the story, and later faded. Students compare across problems (Rittle-Johnson & Star, 2007) and contrast between solution steps on which they succeeded and failed (Walker, Cheng, & Stigler, 2014).

The other two conditions (‘*Purpose-Influenced Formalisms-First*’) were matched to the first for system-of-equations problems and methods. These materials were structured to parallel traditional introduction of variables with formal definitions and lessons followed by student practice. We term these ‘Purpose-Influenced’ because they still provide sixth graders with early systems-of-equations training that may convey a sense of purpose through problem structure. One version was matched in video content (stories) and style to the Purpose-Driven condition, and the other used analogous Khan Academy videos as a baseline. To control for time-on-task and number of practice problems, two versions were run: Experiment A held time-on-task constant across conditions, and Experiment B the number of problems.

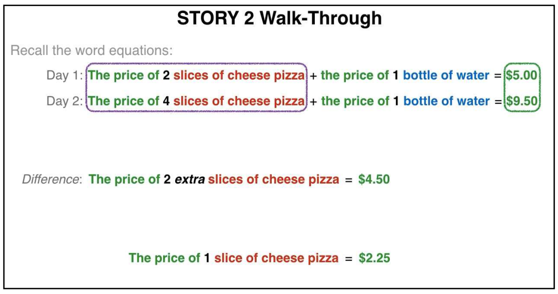

Figure 2. Screenshot of ‘intuitive elimination’ (what changed between two days’ orders and costs) using word equations as a ‘bridge’ representation between sentences and formal variables (PDPF).

If understanding the purpose for algebraic representation is the key missing component in algebra learning, we predicted all conditions (especially PDPF) would outperform baseline groups who did not learn using our approach.

Experiment and results

We recruited sixth-grade participants from two K-6 schools (lab school in Los Angeles County, CA; public school in Riverside County, CA) and randomly assigned them to the three conditions. Respectively, 73 and 90 participants completed our procedure in Experiments A and B. There were three sessions of 1-1.5 hours; the first two consisted of the learning materials and practice questions, 1-2 days apart, and the third consisted of the posttest. Overall, sixth graders received 2-3 hours of learning and a posttest 2-12 weeks later (delay varied by class, equally across groups).

For a baseline comparison, we gave counterbalanced subsets of the transfer items to eighth and ninth graders from the middle and high schools that our Riverside County sample will attend; the 12 eighth graders had learned systems of equations 2 weeks prior via traditional instruction, and 20 ninth graders were tested 2-3 weeks prior to reviewing systems and again 2-3 weeks after. The eighth graders who took subsamples of the posttest scored an adjusted average of 0.58 out of 6, while the ninth graders averaged 2.76 out of 6 before review and 3.86 after review.

The sixth graders averaged correct answers of 3.30 (PDPF), 2.93 (Purpose-Influenced), and 3.16 (Khan Purpose-Influenced) out of 6. We found no significant effects of condition (p’s > .34). Importantly, sixth graders in all conditions performed descriptively better than the eighth and ninth graders pre-review.

Our findings provide suggestive evidence that sixth-grade students can learn system-of-equations problems designed to enhance understanding the purpose of variable representations. The 2-3 hour training enabled them to solve problems at a level comparable to samples of 12 eighth graders post-learning and 20 ninth graders pre-review. Our approach would benefit from follow-up studies with larger samples and more diverse learners.

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