Learners’ Emergent Designs for Play: Game Design as Mathematical Modeling Practices

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Abstract: In this paper, we explore how collaboratively redesigning a game could support a modeling-based view of mathematics learning. We rely on a complexity perspective to structure learning environments that aims to expand collective learning opportunities while valuing the diversity of individual ideas. We explored how we could support learners to come up with their individual ideas on meaningful interactions in possible game systems and to collectively use mathematics to model those systems. We argue that collaborative game design provides opportunities for students to go beyond practicing computation and solving linear problems. They could develop a view of mathematics as a tool to make sense of systems, and experiment with and design new possible structures (i.e., in the modeled world of their game). Our findings highlight the importance of the classroom structures that support learners to develop shared goals while enabling them to pursue diverse paths of engagement with mathematics.

In this paper, we discuss a study on redesigning a board game for mathematics learning. Taking a design-based research approach, we worked with grades 3 and 4 students in a Western Canadian public school. Viewing mathematics as a means for interpreting and modeling systems (Lesh & English, 2005, Davis, 2018), mathematics learning is not merely about practicing computation and solving already mathematized problems that use linearizing assumptions (e.g., limiting the range of inputs). Modeling-based mathematics learning would use mathematical constructs as a tool to model patterns and future possibilities in making sense of phenomena that shape our lives (e.g., population growth). In this paper, we describe how game play and design activities could mediate such engagement with mathematical thinking.

We investigate the students’ group work on redesigning board games, through a complexity perspective to designing learning environments (Davis & Sumara, 2006). In complexity perspectives, learning emerges through the interactions of alternative interpretations. Scholars encourage educators to create conditions open to the emergence of different forms of learning in the classroom (Davis & Sumara, 2006). Learners should engage in common experiences, such as shared projects, tools and materials to support their communication. Such settings should, at the same time, involve tasks that invite different ways of contribution, encouraging learners to express their diverse idiosyncratic ideas. Using this complexity framing, we explore how the students were engaged with mathematical thinking in communicating their ideas, collaborated to create rules of interaction for a working system (i.e. a game), and provided unique possibilities of play for themselves and others. This paper specifically investigates: (1) how the collaborative game redesign approach used in this project provided the structural conditions through which the students communicated their ideas and developed shared goals and understandings; and (2) how the groups’ varied ways of using mathematical constructs in creating systems of meaningful interaction and play unfolded through the project. In the following, we elaborate on our view of mathematics learning and the design of learning environments from a complexity perspective.

A complexity view of designing learning environments

Complexity research explores how diverse rule-following entities (e.g., living organisms) interact and shape a collective whole (i.e., a complex system such as an ecosystem). These entities, or agents, learn, adapt, evolve and form a system, which shows emergent behaviors and collective patterns without benefiting from a central controller (Mitchell, 2009). Complexity scholars in education study the complementarities of individual sense-making and collective understanding as complex systems (Davis & Sumara, 2006). This line of research suggests that individual understanding emerges out of the interaction of sets of ideas within the context of language, culture and, social relations. Such emergence, i.e. individual and collective learning, cannot be directed by pre-set goals. Certain conditions, however, can be conducive to the emergence of new learning possibilities. Learning designs informed by this view have the conditions of emergent learning systems that entail a balance between structure to constraint the vast possibilities, and openness to enable diverse responses. These conditions shape the notion of *enabling constraints* (Davis & Sumara, 2006), rooted in a view towards emergent systems that are “rule-bound” and constrained, but are also flexible and capable of “unanticipated possibilities” (Davis et al., 2015, p. 219). Creating such structural conditions, cannot be distinguished from the educators’ view towards disciplinary practices. In particular, mathematical competency has been widely supported through routine practices of solving linear and defined problems in schools (i.e., established mathematical models of a situation, such as how to evenly spread soil in an area). Therefore, the focus has been on computation rather than using mathematics as a modeling tool (Davis, 2018). Shifting the focus to the latter could open the space for learners to playfully experiment with their own assumptions and the mathematical constructs to make sense of real-world phenomena. We, therefore, suggest that the opportunities to use mathematics to create rules and patterns support learners to express their individual ideas while having more possibilities for their ideas to interact with those of others toward collective learning.

Mathematics as a tool for making sense of and modeling systems

We follow a view of mathematics as a tool for interpreting and modeling systems, phenomena and experiences (Lesh & English, 2005; Davis, 2018). In addition, we view mathematics as a means to mediate learners' imagination and design of new possibilities. When viewing mathematics as the study of structure, and as a tool for making sense of complex systems that shape our everyday lives (e.g., economic systems and communication networks), supporting mathematics learning should go beyond providing the linear path of reaching goals from givens and practicing computation (Lesh & English, 2005). Engaging with non-trivial problems requires "a series of modeling cycles in which current ways of thinking are iteratively expressed, tested, and revised; and, each modeling cycle tends to involve somewhat different interpretations of givens, goals, and possible solution steps" (p. 489). When making modeling the focus of school mathematics, mathematical constructs will be used in the service of interpreting situations and phenomena, i.e., within the modeling activity. In a modeling activity, computation is not the center of learners' mathematics practices, but "a means to an end" (Davis, 2018, p. 84).

Studies have highlighted the potential of board games in developing mathematical competencies (e.g., Saxe, 1992). Games are rule-based structures, in which players act on the representations of a modeled world. Following a view of mathematics as a tool for interpreting and modeling systems, we not only used game play, but also proposed collaborative redesign of board games as the activity that could support learners' mathematical thinking. We argue that redesigning board games could provide unique opportunities for learners to explore the structure of pre-existing games, and beyond that, to imagine and experiment with new possible structures to create meaningful situations for play. Such experimentation would be mediated by the different tools and materials (e.g., games' symbolic pieces, board, and dice) that students use in creating players' resources and rules of their interaction and progress in the modeled world of the game. Through creating game components, learners engage in iterative problem solving that involves number and spatial sense (Saxe, 1992; Barta & Schaelling, 1998), i.e., using measurements, shapes and transformations (e.g., in creating game pieces, board and scoring system), and probability problems of chance and uncertainty. In designing games for mathematics learning, then, mathematical constructs and calculation for precise measurements would be used by students, but only as a means to entertain and communicate various imagined possibilities (Davis, 2018; Nemirovsky and Ferrara, 2009).

We view this approach of using mathematics as a tool to inspire new ideas, as complementary with the complexity-informed view of designing learning environments. Importantly, enabling constraints pertain to keeping a balance between the *coherence* that supports the purpose and identity of the collective and the existing *diversity* that allows the collective to express its creativity and respond to new circumstances (Davis & Sumara, 2006). The commonalities of students' practices (e.g. aiming to use mathematics to model systems in their games) enable the interactivity of individual ideas. Specifically, the goal of redesigning a board game with mathematical constructs is a constraint, but also a basis or platform for collaboration and interaction of students. At the same time, the iterative process of designing and playtesting games not only encourages students' diverse ideas but also helps them reinterpret and modify their assumptions and goals. This process can potentially enable the variations in learners' designs and the accompanying learning possibilities.

Research approach and data sources

The study is part of a design-based research project (Collins, Joseph, & Bielaczyc, 2004) on developing a game design-based approach to learning. Based on the result of the first iteration, which showed difficulty in designing board games from the scratch (Kim & Bastani, 2017), this second iteration was designed for Grade 3/4 mathematics learning through redesigning an existing board game. It was conducted in two to three forty-five-minute sessions per week, over six weeks. We collected ethnographic data through field notes, video-recordings of the classroom and photos of students' in-progress and final game designs.

Project context and findings

The collaborative game design approach used in this project involves using multiple modes and materials for communicating and negotiating ideas and transforming them into shareable artifacts (Kim & Bastani, 2018). By choosing signs, symbols, and rules, learners could create coherent systems (i.e. games) that set multiple situations for players' participation in motion. In this study, all the groups started with the shared goal of creating a 2-dimensional (2D) version of the game *Inversé* and went through cycles of playtesting to balance their game. The students' designs, however, unfolded in various ways, differentiating the final games in terms of components, rules, number of players and the degree to which they relied on players' chance or strategy. Below, we discuss the project structure, which provided common grounds for the students' communication within and among the groups, to describe the classroom context. We also discuss the commonalities at the group and class levels that were co-constructed through iterative practices, and how they extended the students' shared understandings and exploration of possible designs (Kim, 2018). We then describe our findings on students' emergent co-designs. We discuss how their ideas were expressed, evolved and converged into shareable artifacts, and the ways they used mathematical constructs to create play situations in their unique games. We will specifically provide examples from the interactions and designed artifacts of two groups in this classroom, namely *Extreme Versé* and *Markit.* The teacher's and students' names used are pseudonyms.

Commonalities as infrastructures for communication and idea development

The project shared goals

Creating a 2D version of Inversé. Before starting the project, the students played multiple games and with their teacher, Ms. Gagne, chose Inversé for their project. Inversé, as a two-player game, has a set of five different wooden blocks with the same volume but different dimensions for each player. Players take turns to place their pieces until one of them can no longer fit a piece. Its rules include not touching same-color pieces, not placing same-color pieces in the same orientation, and not touching same-height pieces (Figure 1). In each turn, the player should consider possible positions of their 3D pieces to fit them on the board while making the next move more difficult for the opponent. It, therefore, could engage learners with the concepts of volume and area (as they try the blocks' different faces on the game grid board), rotation and estimation. In redesigning Inversé, learners were expected to explore the game structure and transform the components and rules to create a game with 2D pieces. Initially, the teacher asked the students to think about the possible designs individually, and then had them share their ideas in groups and integrate them into one game. She emphasized listening to others' viewpoints and being ready to change the design ideas in this process. The common starting point, i.e., playing Inversé, understanding its rules and aiming to change it into a 2D game, and using simple materials (grid papers, scissors and markers) facilitated the communication within the groups. It also helped them easily make sense of other groups' designs when shared.

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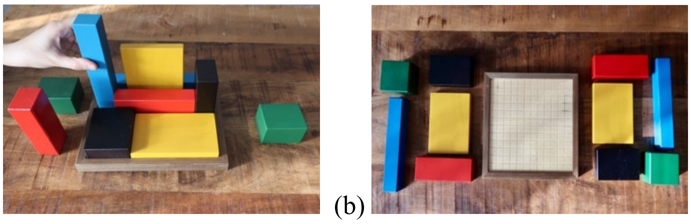


Figure 1. Inversé: (a) the game pieces (b) a possible play situation based on the rules.

Using multiplication and estimation. In the subsequent sessions, Ms. Gagne discussed developing math skills in multiplication and estimation as one of the goals of the students' games. She asked the students to decide about their game components (e.g., players' pieces, dice, and board) and their size, and how they want to use them in their game.

One group (Sajan, Avani, and Talia), who later named their game *Extreme Versé*, started with cutting and coloring different rectangular pieces (Figure 2, green pieces) and making cards that showed two numbers (Figure 2, white paper pieces with numbers). Sajan explained to Ms. Gagne (Figure 2, top right corner) that the numbers on each card showed the perimeter of a rectangle (although there were two numbers on each card) and that players should estimate the rectangle's area based on the perimeter and choose the right one among the pieces. Each turn, players would randomly pick one card and choose the right piece and put it on the grid game paper. The player who could fit all their pieces on the board would win. Ms. Gange complimented them for having a good starting point for discussing the concepts of perimeter and area. However, it seemed that there existed a confusion about the concepts of area and perimeter in the group. Using Inversé's original, grid-like game board, she showed an example of a block covering a 3x2 area (the red block). She then drew a 3x2 rectangle on a grid paper and wanted the students to explain how they could find its perimeter and area. It was primarily to challenge them to distinguish between these two concepts. Sajan gave correct answers and the teacher wanted all the group members to work more on the same example. She also asked them to explain again what the two numbers on their cards mean. Avani mentioned the numbers specified the width and length of the rectangle that players should pick. This was different from Sajan's initial explanation. Ms. Gange suggested that they could also create a card for each piece specifying its area and perimeter. Ms. Gagne visited different groups and went through a similar process of discussing area estimation for creating their game pieces.

Figure 2. The group discussing the concepts of perimeter and area with the teacher.

Co-constructed commonalities

We observed that the initial intentional conditions that the project started with (i.e., playing a specific game and redesigning it with a focus on using the constructs of multiplication and estimation) stimulated certain kinds of discussions and iterative practice. This helped the communication at the group and class levels and extended the learning possibilities for the students. We mainly focus on two emergent themes: the shared expectation of designing balanced games and the shared understanding of the design process as iterative and collaborative.

The shared expectation of creating a balance between chance and strategy. As the students implemented different design ideas, we observed that they started having deeper discussions on their designed rules, signs, symbols (e.g., numbers) and their game components. The Extreme Versé group discussed using the number cards they had initially designed (players would randomly pick one card to choose the right rectangular piece and play with):

> Sajan: Now I kind of feel removing the cards, because it more tells you what to do. It doesn't show you your thing.
>
> Talia: [But] we need some randomness.
>
> Sajan: We do not need randomness. How about without randomness?
>
> Avani: Why did you make these cards? [...] Without the cards it would not be a math game. It would be take a piece and place it, take a piece and place it.

Such conversations around how to make their games based on players' decisions versus chance as well as what it means to use math in designing their games (as observed in Avani's comment) also emerged among other groups and continued throughout the project. Ms. Gange, who frequently plays board games outside the school context, also focused on how games should provide situations for meaningful decision making and strategizing, rather than testing calculation skills or just how lucky a player is. Another group, *Markit*, came up with the idea of using two dice for players to draw rectangles on a grid board and to use multiplication. The numbers on the dice determined the rectangles' widths and lengths. Players had to find a free space on the grid board to draw their rectangle. In a discussion with the group, Ms. Gange acknowledged that their idea of using dice rather than prepared rectangular pieces would push players to think of the area as they drew rectangles. She, however, challenged the group to think about how they could change their game components "to make their game more complex", requiring players to make harder decisions. This conversation stimulated a discussion on how changing the board size or even using dice with bigger numbers (Figure 3) and adding rules would create more challenging situations for players. We observed that a common language, which involved "strategy vs. chance", "not creating knowledge testing games" (for calculation skills), "estimation", "decision making" and "randomness", emerged out of the class- and group-level conversations. The shared expectation of the games being more based on strategy as opposed to chance supported the students' communication and idea development in their group playtests as well as when they provided feedback on the other games. Importantly, it opened the space for the students' further exploring how using different components and mathematical constructs (e.g., area) could lead to different play situations, which we will evaluate further.

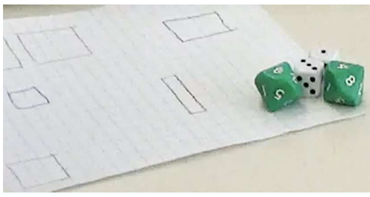


Figure 3. Trying the game ‘Markit’ with different size dice.

***Understanding game design as an iterative and collaborative process*.** As described above, from the beginning, the students were encouraged to try different design ideas and change them as they tested their games. Playtesting, then, was a significant part of the students’ work and essentially engaged all the group members. Creating a playable game is about anticipating different situations and opening the space for even unexpected strategies, which differentiate game design from other kinds of design projects (Kim & Bastani, 2017). Other groups, as well as the students from another class, also playtested the games. This helped the groups have new design ideas to improve upon their games (Figure 4). We observed how playtesting and encountering unexpected outcomes gradually shaped a shared understanding of game design as a dynamic and collaborative process.

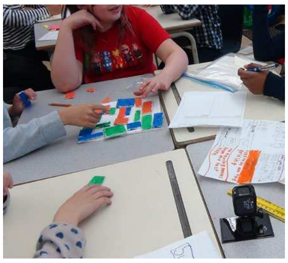
(a)  (b) 

Figure 4. Other groups’ playing and giving feedback on the games Markit (a) and Extreme Versé (b).

In our interview with the Markit group (Elene, Aaron, and Natalie), Elena described game design as challenging while also emergent and fun: “because you have a game but then you have to make a different version of it, so then it’s also kind of fun because one day you might just become famous for making a new version of a game.”

Emergent design ideas and mathematics practices

The commonalities in the classroom, which acted as structured or emergent frames, supported the groups and the class to maintain their collective purpose and coherence, and facilitated the communication of students’ individual ideas. In addition, we view these commonalities for supporting students’ interactions necessary for the emergence of design variations in the open-ended task of creating games. In this study, we explored how the process of game design and playtesting opened the space for students’ reimagining possible interaction situations and, consequently, for variations in their artifacts and use of mathematics. Although all the games used a grid board and rectangles, the final games were all unique, in terms of the number of players, how players picked rectangles (using dice or prepared pieces), the rules for placing or drawing rectangles (e.g., if and how the rectangles could touch), and the rules for winning (e.g., by trapping the opponent’s rectangles or by fitting the last piece). To evaluate the ways the students used mathematics in designing their games, we will focus on two main categories that all the games could fall into: having prepared rectangular pieces or using dice for drawing pieces. We specifically found Extreme Versé and Markit as good examples of the designed games in these two different categories. Using these examples, we will discuss individual design ideas leading to emergent design decisions in the groups in the process of learners’ collaborative game design.

Diverse individual ideas about the games’ features

In the process of group game design, we observed how individual students’ diverse ideas, based on their interests in particular game features, triggered a sequence of design decisions and use of math in each group. After revising their game’s first version, described above, Sajan (from Extreme Versé) suggested that they design a three-player game, each having the same set of pieces consisting of prepared rectangles in different sizes. To specify each player’s pieces, they decided to paint each set with a different color (Figure 5). This, as he explained, differentiated their game from the original game, Inversé, being a two-player game and having blocks with various colors for both players. Talia and Sajan cut six different rectangular pieces for each player with different sizes (some were adopted from the Inversé’s blocks, and others were randomly determined). After discussing how they could come up with their board size with the teacher, Sajan decided to calculate the area of each piece and to sum the areas up. After doing “the biggest addition problem in this world”, as Sajan put it, he came up with the number 2166 as the area of the game board they needed. Although it seemed to Talia that it would be too big for their game board, she and Sajan started to glue some grid pieces to create the board and try it.

Figure 5. Extreme Versé: creating three sets of pieces.

The group Markit started with Elena’s suggestion of using both dice and prepared pieces in their two-player game (Figure 6-a). Players had to roll dice and then find the piece whose sides matched with the numbers on the dice. They prepared 4 pieces for each player (which didn’t cover all the possible combinations of numbers on their two 1-6 dice). When one of the researchers asked if the last person who could fit a piece on the board would win, Aaron explained, “like there are a lot of squares here (showing a grid board) and I got 2x2. So, if I put it here (drawing a hypothetical square by his finger in the center of the board), you will have no space anywhere.” They discussed how smaller or bigger boards could make players’ decision about where to draw pieces more difficult or easier. After trying their initial idea of using both dice and prepared pieces, they moved to Aaron’s idea of using only dice and drawing pieces based on the numbers on the dice on their grid board, rather than using prepared pieces (Figure 6-b).

Later, in our interview with the group, Elena talked about how she liked the idea of using dice. She explained that it differentiated their game from the games with a similar win condition (i.e., fitting the last piece on the board), that used prepared pieces for players to choose from. She described starting from the biggest piece as an easy access strategy for games with prepared pieces, as small pieces could fit more easily on the board at the end of the game: “I like the dice ...you always want to get the smallest pieces, but if you didn’t have the dice and you got to choose, then you’d always do a one by one [, from the biggest to the smallest piece].” This showed how she saw the randomness the dice added, could make the decision making for how to fit the pieces on the board more challenging. The variations in the individuals’ ideas about the games’ features (e.g., number of players, players’ pieces and play rules), therefore, engaged them with different mathematical practices (e.g., use of area, multiplication, estimation, and randomness). Through these mathematical practices, the students applied, shared and tested their initial design ideas and went through a subsequent design decision for their games, which were further developed with their groups.

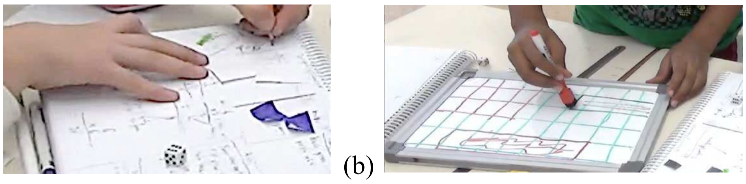
(a)  (b)

Figure 6. (a) Elena’s idea of using both dice and prepared pieces, (b) Aaron’s testing with dice and drawing pieces.

Co-exploring possible game designs and play situations

The groups collaborated to incorporate different individual ideas and experiment with variations of their game pieces, boards, and rules. It led to co-exploring more challenging interaction situations and win conditions. The iterations of design, testing, and problem solving contributed to the emergence of unique features and variations in games.

The Extreme Versé group had difficulty with the size of the game board. They playtested their game with the big board they had made, with one of the researchers (Figure 7). In this later version of their game, players placed prepared pieces on empty spaces on the board in a way that same colors (i.e., a players’ own pieces) and same shapes wouldn’t touch. The last person to fit a piece, would win. During the play, Talia suggested they should have more pieces as the game was ending very quickly. After the playtest, they also had a conversation with the researcher about their board being too big, which made fitting the pieces very easy. With their relatively big-sized board, there could be a tie situation, in which all the three players could place all their pieces on the board. The players would have too many empty spaces to strategize and estimate carefully to fit pieces. Talia came up with the idea of creating two game boards, with a smaller board for a harder version of their game and a bigger one for a basic version. As they were asked to come up with only one game, they finally decided to revise their current board so as to accommodate more meaningful decision making for players. Their method of creating the right size for their board was through testing smaller boards. They gradually cut down their board and tested how the game became more challenging, requiring careful estimation of the area to fit the pieces. We therefore observed that creating a more complex game led them to go beyond understanding their design challenge as a mere linear computation problem, unlike how Sajan had initially perceived in determining the board area. The mathematical thinking in this design project involved going through cycles of modeling ideas and exploring possible solutions, getting feedback and adapting the designs with their peers.

Figure 7. Extreme Versé: playtesting with the big size board.

The Markit group had the challenge of creating a balance between chance and strategy. Their first version only needed players to roll dice to determine the length and width of their rectangular piece, draw the piece where they could find estimated spaces, and be the last one to fit a piece. Through playtesting with their teacher, they came up with the idea of using a smaller board or dice with bigger numbers, but they found this wouldn't require players to use more strategies. Elena suggested adding one rule to the game (Figure 8-a): "What if the pieces have to touch other pieces' corners [and only their corners and not their sides]?" This idea added to the complexity of their game requiring players to strategize, based on the board condition and their random situation in each turn. It needed players to engage with mathematical thinking, including more conscious estimations and mental rotations. To make their game even more challenging, they tried different versions of rectangular game boards. In the end, however, they decided to use a square shaped board that provided more possibilities for drawing pieces that only touched corners of already-drawn rectangles (Figure 8-b). They then faced an optimization problem (i.e., deciding on optimized board's dimensions for this game) which encouraged them to test their game with various board sizes and shapes. The ongoing discussions about making a balance between chance and strategy, not only stimulated a common language for exchanging design ideas, but also engaged learners with more complex modeling problems. It subsequently triggered more unique collective design decisions and therefore more variations in games. Talking to researchers about how simple games could become more complex through the design process, Elena used a remarkable analogy: "Kind of like a book, so it starts out with a word then it becomes sentences, then pages and then chapters."

(a) (b)

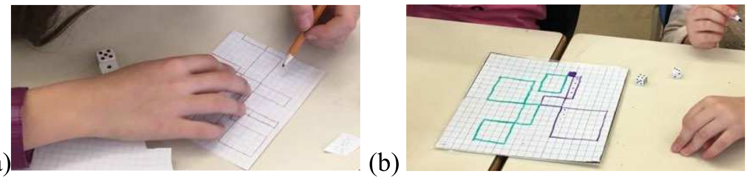


Figure 8. Markit: adding the rule of corners' touching (a) and the final game (b).

Discussion

This study contributes to supporting mathematics learning through modeling a system from a complexity perspective. We proposed collaborative game design projects for students to communicate their diverse ideas and use of mathematical constructs to create interaction and play situations. Informed by a complexity perspective (Davis & Sumara, 2006), our analysis showed how co-designing games could provide a platform for learners to pursue shared goals through which common design languages emerged. At the same time, it opened the space for the emergence of variations in the students' designs and use of mathematics. We specifically attended to how the process of game making encouraged the students to embrace their teammates' diverse perspectives and the feedback from the teacher and other groups. Through making decisions about game rules and meaningful play, they incorporated mathematical ideas and developed new insights as to what constitutes mathematics. This was exemplified in an occasion of providing feedback on the game Extreme Versé: the students from two groups expressed their different interpretations of what it means for a game to involve mathematics. While one of the students believed that there is no math in the game without any explicit calculations, other students argued that there was math in the game when players would think where to put their pieces, as players' decisions involved estimation and an understanding of area. We believe this project provides insight as to how the view towards disciplinary knowledge and the practice of design for learning could reinforce each other. Perceiving mathematics learning as the mastery of skills imply a one-way relationship between the learner and knowledge, satisfying predefined curricular goals. Viewing mathematics as a tool to interpret phenomena and model systems, however, would reinforce the design for mathematics learning that allow for the participation of diverse ideas and new interpretations of systems around us. This would engage learners in using mathematical constructs to create meaningful systems that can be shared with others. Our findings also exemplify the potential of forming small size groups, as well as communicating within a larger scale network of groups. Working on a shared project allows the small groups to maintain their coherence and the individuals to express their creativity and feel their agency in their shared projects. At the same time, the anticipation of sharing their designs with others outside their group would encourage them to seek multiple viewpoints and design possibilities. Based on this study, we also argue for incorporating open-ended tasks, with designed constraints (mainly at the starting point), that can become increasingly more complex. The constraint of playing Inversé and having the shared goal of redesigning it limited the focus on possible systems to be designed and mathematical constructs to be used. At the same time, these constraints opened the ground for mathematically rich conversations around the original game system and novel possible models. The collective design ideas unfolding through the cycles of design and playtest were intertwined with the emergence of co-constructed commonalities, as developing constraints (e.g., making their game balanced and incorporating specific game features), which stimulated specialized paths of using mathematics in groups.

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