Math: It's Not What You "Think"

José F. Gutiérrez, University of Utah, jose.gutierrez@utah.edu

Abstract: This paper presents an emerging pedagogical approach for implementing critical mathematics. I describe a specific tactic for teaching mathematics: to avoid using the word "thinking." This approach is informed by scholarship on race, critical mathematics, and embodied cognition. I argue that using the term "thinking" is not necessary for facilitating authentic learning experiences. Moreover, this tactic disrupts a number of pernicious historical narratives (i.e., stereotypes) about mathematics education and about who can and who cannot do mathematics. The crux of the paper is that avoiding the word "thinking"—and other terms associated with "smartness"—loosens the grip Cartesian epistemology has on mathematics education by shifting classroom discourse away from math-as-*thinking* to math-as-*action*, which may, in turn, create stronger opportunities for learning. From a learning-sciences perspective, I present other hypotheses stemming out of this pedagogical approach that warrant further research.

Keywords: Cartesian dualism, embodied cognition, equity, mathematics learning, race

"Who thinks that they could explain their thinking for question one?"
– Deborah Ball to her fifth-Grade math class

"You should have those five Gauss-Markov assumptions sort of… in your head."
– Statistics professor to his graduate students

"There is no right or wrong answer. We're only interested in your thinking. … You've done a fantastic job of explaining your thinking!"
– Mathematics education researcher to a middle school student

"What do you think?"
– Everyone

Introduction and objectives

Thinking and related phrases, such as *What do you think?*, are ubiquitous in secondary math classrooms (Herbel-Eisenmann, Wagner, & Cortes, 2010). They are assumed to be pedagogically neutral or positive. This paper is part of my ongoing research that challenges and problematizes this assumption.

Focusing on "thinking" and related notions like abstraction and manipulating symbols has the potential to overlook other authentic forms of knowing in mathematics. In a previous study (Gutiérrez, 2015), I conducted a year-long participatory ethnography of a secondary math class. I focused my analysis on Mr. Matthew Lam, a teacher who believed that "math=thinking=liberation." He repeatedly presented this metaphorical "equation" to his class, and its underlying logic appeared to guide his instructional decisions. One of Mr. Lam's primary pedagogical objectives was to support student thinking—*critical* thinking in particular. He associated certain types of mathematical thinking (e.g., generalization) with the ability to "see," "critique," and "understanding systems," which is something that he sought to foster in his students as part of his social justice pedagogy (Gutiérrez, 2015). That said, what Mr. Lam called mathematical "thinking" emphasized "abstractions" that involve "letters." In his instruction, Mr. Lam occasionally conflated mathematical reasoning with using symbols. This negatively influenced his students' agency, because it overlooked other authentic forms of knowledge and knowing that did not rely on "abstraction" or manipulating symbols as evidence of mathematical "thinking." For instance, some students constructed generalizations using an array of semiotic resources such as verbal speech, rhythm, gesture, repetition, and non-conventional notations that were never sanctioned by the teacher (Author, 2016). If students' emergent ways of knowing and different forms of knowledge go unrecognized, this can lead to distorted notions of what counts as mathematics.

Mr. Lam's expressed belief that "math=thinking" is not unusual. In a linguistic analysis of 148 classroom transcripts involving eight different math teachers in a variety of school contexts, Herbel-Eisenmann, Wagner, & Cortes (2010) reported that the second most frequent group of words found in their data was *What do you think*. This particular phrase, which they refer to as a "lexical bundle," had 198 instances spread across 8 classrooms. (The most frequent bundle was a directive, *I want you to*, which had 333 instances spread across those same 8 classrooms). These findings indicate that notions of math-as-*thinking* are quite common in classroom discourse. The objectives of this paper are: (1) to argue that this hidden aspect of mathematics education is not just a semantic issue but a substantial point that could have a tremendous impact on student learning; (2) to describe a specific tactic for teaching mathematics: avoid using the word "thinking" when working directly with youth in mathematical contexts; and (3) to present a series of hypotheses stemming out of this pedagogical approach that warrant further research.

The Cartesian Grip on mathematics education

The use of the term "thinking" is not necessary for facilitating authentic learning experiences and fostering conceptual understanding. On the contrary, the constant use of this term in classroom discourse reinforces a Cartesian mind-body dualism (1) that *interferes* with learning mathematics (cf. Cook & Brown, 1999), as in the case of Mr. Lam and his students (Gutiérrez, 2016). Functioning just under the radar, the Cartesian view of mathematics privileges knowledge "possessed" or "acquired" by the individual over group participation and social practice (cf. Sfard, 1998), which severely underestimates the importance of social interaction in conceptual learning (Radford, 2003; Sfard, 2007; Vygotsky, 1978). Cartesian dualism also promotes stereotypical notions of *mathematics* based primarily on analytic, decontextualized reasoning which, in turn, promote false notions of *mathematics education* as an *a*cultural and *a*historical enterprise. Critical educational scholars have repeatedly argued against the idea of mathematics education as politically neutral because it ignores the structural dimensions in the organization of learning (Nasir, Hand, & Taylor, 2008; Valero & Zevenbergen, 2004).

To clarify, this paper *does not* argue that thinking is absent during mathematical activity or problem solving. What I am arguing, is that there are social-historical narratives that *equate* mathematics to thinking (i.e., "math=thinking"). Moreover, these narratives posit that *pure thinking* (i.e., mental activity that is disconnected from perceptual or bodily activity) is what drives mathematical discovery and therefore learning. The Cartesian view of mathematics reproduces a stereotype of mathematics as *tantamount* to pure thinking. Math learning is believed to be mostly an "amodal" phenomenon, a way of reflecting on the world that begins and ends "in the head." The *process* of mathematical problem solving is to "think about stuff," and the final *products* that result from that "pure mental activity" are other clearer/logical/cogent ways of thinking. An exclusive focus on math-as-thinking ignores the fundamental roles that the human body, action, discourse, and signs play in mathematics learning (Abrahamson & Trninic, 2015; Alibali & Nathan, 2012; Nemirovsky, 2003; Wertsch, 1998).

Furthermore, this problematic stereotype of math-as-thinking is fundamentally linked to another socially constructed narrative: math-as-*intelligence*. Through the prism of Cartesian epistemology, mathematical ability is often seen as an index of general intelligence, rather than simply the development of skill or competence within certain genres of semiotic activity. In their analysis of race and mathematical practices, Shah and Leonardo (2017) further unpack the notion of math-as-intelligence and point to one of its limitations:

> The practice of doing school mathematics in the U.S. context is associated with a particular set of meanings. Mathematics is typically viewed as the most difficult of all subjects; only the elite few are thought to possess the innate capacity to understand mathematics (Schoenfeld, 2002). Perhaps as a result, mathematical ability and intellectual capacity are believed to go hand in hand (Ernest, 1991). And because mathematical ability is viewed as innate, it tends not to be seen as something that can develop through concerted effort over time. (p. 60)

In sum, the deployment of the word "thinking," explicitly in mathematical contexts, functions as an insidious vessel that laces everyday classroom discourse with Cartesian epistemology. In turn, classroom discourse that subscribes to Cartesian precepts overlaps with racial-mathematical discourse (Shah, 2017; Shah & Leonardo, 2017) that serves to create a "racial hierarchy of mathematical ability" (Martin, qtd. in Shah, 2017, p. 11). (2)

Mathematics education research as complicit with the math-as-thinking regime

Through the use of everyday terms such as "thinking," Cartesian dualism is "baked into" the everyday language of mathematics education. This results in the reproduction of hierarchical category systems and phraseologies related to both mathematical ability and intelligence (e.g., "fast" versus "slow kids"; "advanced" versus "below basic" courses; and "failure" versus "success" with problem-solving) that have pernicious effects on learning and student identity (Horn, 2007; Larnell, Boston, & Bragelman, 2014; Ruthven, 1987). Furthermore, the implications of these hierarchies extend beyond the classroom. I maintain that constructs stemming from the research world—such as "grit," "smartness," and "resilience"—are actually derivative of Cartesian dualism.

Whereas certain research constructs are intended to be useful tools for investigating and improving mathematics education, they in fact "tighten" the grip and delimit opportunities for learning, because these, too, are *relational constructs* (Shah, 2017) based on "differences" that have been demarcated, historically, along racial lines. Therefore, these constructs, although believed to describe "objective" properties of individual students so as to identify leverage points to bolster "smartness," for example (Leonardo & Broderick, 2011), instead serve to reproduce and strengthen the racial–mathematical hierarchy due to the tight, three-way link between mathematical ability, intelligence, and race.

Future research

To loosen the Cartesian grip on mathematics education, I suggest eliminating certain types of statements during classroom discourse (see Table 1). I hypothesize that students would respond differently to prompting strategies that emphasize action and perception (cf. Abrahamson, 2012). A set of specific hypotheses stem from this approach (Table 2), which future research can explore in laboratory settings or classroom contexts, or both.

Table 1: Two different prompting strategies

Avoid these prompts:	Use these instead, focus on perception and action:
What do you think?	*What do you* ***see****? (Alt: What do you* ***notice****?)*
How are you thinking about that?	*How are you* ***seeing*** *that?*
Can you explain your thinking?	*Can you show/explain what you did?*
How do you think THIS is related to THAT?	*How is THIS related to THAT?*

Statements such as those in the left column of Table 1, above, reify Cartesian epistemology and stereotyped notions of mathematics that ignore the bodily activity. Statements from the right column (along with other discursive action such as gesturing, body movement, and voice prosody) shift the attention to perceptual and sensory-motor action that can provide the substrate for conceptual learning. Avoiding explicit mention of *thinking* in classroom discourse, and instead drawing on other semiotic and embodied measures, can shift the discourse away from math-as-thinking to math-as-action, which creates more robust opportunities to learn (Abrahamson & Trninic, 2015; Alibali & Nathan, 2012; Nemirovsky, 2003).

Table 2: Hypotheses to be explored

Hypothesis:	Suggested study and research questions ("RQ"):
1. Avoiding "thinking," as a primary prompting or questioning strategy, will elicit different responses from learners.	Controlled laboratory study. Conduct basic experiments that explore and compare the types of mathematical statements and strategies elicited by math-as-thinking versus math-as-action prompts in a structured setting. Example RQ: *Do "thinking" and/or non-"thinking" prompts support the construction of mathematical generalizations?*
2. For teachers or instructors of mathematics, avoiding "thinking" will lower the threshold to gesture (cf. Kelly, Byrne, & Holler, 2011), possibly improving communication with learners.	Classroom-based study: conduct case studies of teachers' initial attempts to use this tactic. Example RQ's: *Do teachers gesture more? Do they gesture differently? What is the nature of the referents indicated by their gesturing actions? Is the conceptual space (in terms of these referents) expanded as a result of this tactic?*
3. "Loosening the Cartesian grip" is a useful theme for pre-service teacher training programs as well as continuing professional development programs. Teachers can engage in discussion of, as well as explore in their own practice, a variety of discourse strategies whose affordances for teaching/learning are well-documented.	Teaching and teacher education study. Example RQ: *How can we support teacher learning of different instructional models, orientations, and tactics?* A few examples include: • "Professional perception" of mathematical objects and processes (Goodwin, 1994; Stevens & Hall, 1998) • "Revoicing to position" (see Enyedy et al., 2008) • Designing instructional gestures (see, e.g., Alibali et al., 2013) • "Action before concept" (Trninic & Abrahamson, 2013)

Concluding remarks

In general, teachers and instructors of mathematics interested in implementing this approach might find it challenging to make this shift in discourse. That said, avoiding "thinking" can bring about not only a superficial linguistic shift, but can actually spur a shift at the level of deep semiotic-cognitive structure, and thus expand a teacher's communicative means of instruction, including bodily activity. This constraint on what a teacher does (or does not say) can induce critical reflection on what exactly is being communicated and how it's being communicated, in situ, regarding constituent elements in a given problem space. This approach has the potential to introduce precision not only in terms of general language-use, but also in the moment-to-moment multimodal micro-practices that engender shared meaning-making and conceptual learning. Over time, a classroom culture based on this approach could enhance both the teacher's and students' semiotic means of objectification of mathematical objects, structures, and generalizations (Gutiérrez, 2013, 2016; Radford, 2003).

Endnotes

(1) Cartesian epistemology is a branch of philosophy attributed to René Descartes and his study of knowledge. Cartesian dualism refers to his concept of the mind and the body as two completely different types of substances (immaterial vs. material) that interact.

(2) Danny Martin proposes that this hierarchy is one in which "students who are identified as Asian and White are placed at the top, and students identified as African American, Native American, and Latino are assigned to the bottom" (qtd. in Shah, 2017, pg. 11).

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