Constructing Knowledge: A Community of Practice Framework for Evaluation in the VMT Project

Michael Khoo, Drexel University, khoo@drexel.edu
Gerry Stahl, Drexel University, Gerry@GerryStahl.net

Abstract: This paper describes a formative evaluation of the Virtual Math Teams (VMT) Project as it adopts the GeoGebra dynamic-mathematics application. The project team is considered as a Community of Practice, which communicates with student users through boundary objects. An ethnographic action research approach is used to analyze three sources of data: VMT user manuals; screenshots of tool interfaces and assignments; and logs of student chat sessions addressing those assignments. The analysis focuses on how the topic of 'construction' is articulated in each data set. The results show that the team understands 'construction' in terms of a complex web of knowledge of dynamic geometry, while the students develop their own emergent notions of 'constructing' from the boundary objects produced by the team. Recommendations for boundary object design are considered.

Keywords: action research, boundary objects, communities of practice, design research, dynamic geometry, ethnography, GeoGebra

Introduction

A basic distinction in project evaluation is that between summative and formative evaluation (Frechtling, 2002). Summative evaluation assesses the final outcomes of a project, while formative evaluation assesses ongoing processes, focusing on how that project is achieving its goals. Evaluations reported in the literature are often summative in nature, and explicit cases of formative evaluation are infrequent. One issue here is that the ongoing nature of formative evaluation, and the iterative generation of interim data, present a less tidy unit of analysis than summative evaluation. However, formative evaluations are useful, for instance in design research, where projects follow iterative design cycles of development, implementation, and assessment. Where a summative evaluation may conclude that one intervention has better outcomes than another, a formative evaluation may suggest how to adjust the intervention to be more effective. This paper introduces a formative evaluation of work with the Virtual Math Teams (VMT) Project as it adopts the GeoGebra dynamic-mathematics application. One aim of the evaluation is to evaluate the ongoing ways in which the VMT project team is codifying its knowledge of dynamic geometry into the project's online tools and documentation, and how well these tools and documents support students to engage in dynamic geometry. This contributes to the ongoing incremental improvement of the project's artifacts, and is a typical use of formative evaluation (it would be of little use to wait until the project has been completed in order to evaluate these materials). The evaluation approach described in this paper draws on theories of Communities of Practice (Lave & Wenger, 1991; Wenger, 1998) to consider VMT not just as a series of tools, assignments and users, but as an ensemble of people, practices, processes, and other phenomena, which all aim at education in dynamic geometry. The analysis focuses on recent pedagogical artifacts produced by the project, the ways in which these artifacts represent the educational intentions of the project team, and how these artifacts are used by the project's student users. The analysis focuses on the team's understanding of dynamic geometry, the reifications of these understandings in informational artifacts, and the understanding gained by students after interacting with these reifications. An important issue here is the extent to which the design of VMT tools and curricula help students to understand dynamic-geometric construction in the same ways as the project team.

The VMT Project with GeoGebra

VMT is a CSCL project spanning over a decade (Stahl, 2013). The project's technological, pedagogical and analytic components provide an integrated online platform for middle- and high-school students to engage in online mathematical discourse as they explore dynamic geometry. Geometrical construction and explanation are emphasized. In classical geometry, these practices involve the use of a straight edge and a compass. In VMT, these concrete affordances are 'translated' (Stahl, 2013a) into a virtual environment and dynamic screen tools. VMT incorporated GeoGebra for the past several years. The system now includes an online environment with a chat window, a virtual whiteboard, and a range of interactive dynamic-geometry tools, which students use to learn about dynamic geometric construction (Stahl, 2011). The project follows a design-based research approach, with iterative cycles of user-centered design, implementation and evaluation. The dynamic-geometry tools in VMT have been developed over a number of years by a team of pedagogists, learning scientists, coders, discourse analysts, social scientists, HCI experts, evaluators, and others. Key components of the design process are the team's weekly meetings, in which members discuss topics such as the student chat logs, curriculum design, technological issues, and paper writing. Outputs from the meetings include revised assignments, analyses of project data, drafts of papers, and technical bug reports and fixes. These and other outputs feed into the iterative development of the educational artifacts that mediate the project to its users: the online tools, the tool documentation, the curricula, specific assignments, and so on. It is a form of mutual bootstrapping, with implementations of the tools and curricula generating data for the research team, the analysis of which supports improvements in the tools and curricula, which in turn leads to the generation of new data for analysis. The project team's hope is that by following a design-research process, the educational artifacts they produce will be more useful than a 'one-shot' design approach.

Communities of Practice

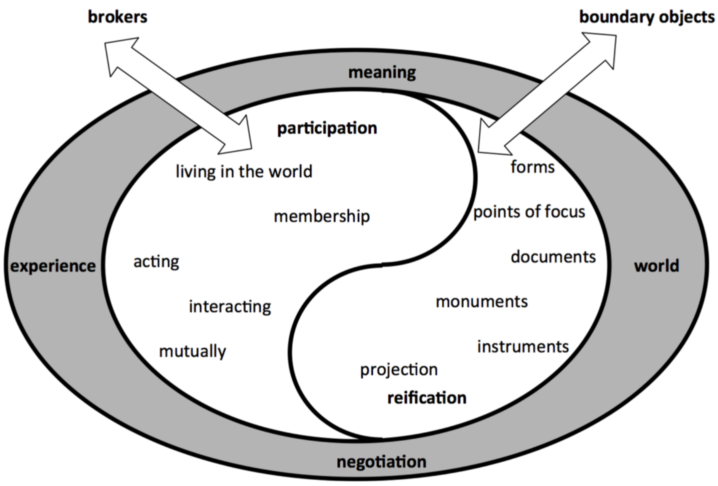

Figure 1. Participation and reification in Communities of Practice. (c.f. Wenger, 1998, Figs 1.1 and 4.1.)

Communities of practice are often glossed somewhat simply as 'groups of people working together on a common task.' The concept has however considerable theoretical depth, including attempting to account for how knowledge is constituted and shared within and between groups. Lave and Wenger (1991) draw on theories of practice by Giddens (1979), Bourdieu (1977), and others, to theorize how newcomers become members of a CoP and gain knowledge of its practices not just by learning what a community knows, but what community members do. This includes knowing *what* to talk about in that community, and also *how* to talk and to support community discourse and memory. Experienced community members guide less experienced members through the community's practices, a process known as legitimate peripheral participation, "an engagement in social practice that entails learning as an integral constituent." Wenger (1998) further elaborates membership in a CoP in terms of a duality of participation and reification (Figure 1). Participation involves "the social experience of living in the world in terms of membership in social communities and active involvement in social enterprises," while reification includes a range of activities (making, designing, representing, naming, encoding, and describing, as well as perceiving, interpreting, using, reusing, decoding and recasting") which generate traces of that membership. The processes are distinct yet mutually constitutive: "[Reification] always rests on participation: what is said, represented, or otherwise brought into focus always assumes a history of participation as a context for its interpretation. In turn, participation always organizes itself around reification because it always involves artifacts, words and concepts that allow it to proceed" [pages].

Given that meaning is constituted internally in CoPs in terms of the participation-reification duality, what might individuals external to a CoP make of the reifications of that CoP? Here, says Wenger, boundary objects play an important role. According to Star and Griesemer (1989), boundary objects are:

> objects which are both plastic enough to adapt to local needs and constraints of the several parties employing them, yet robust enough to maintain a common identity across sites … They may be abstract or concrete. They have different meanings in different social worlds but

> their structure is common enough to more than one world to make them recognizable, a means of translation. The creation and management of boundary objects is key in developing and maintaining coherence across intersecting social worlds.

While CoPs regularly appear in the CSCL literature, boundary objects appear less frequently. The rest of this paper therefore focuses on VMT as a CoP and its use of boundary objects.

Boundary objects in VMT

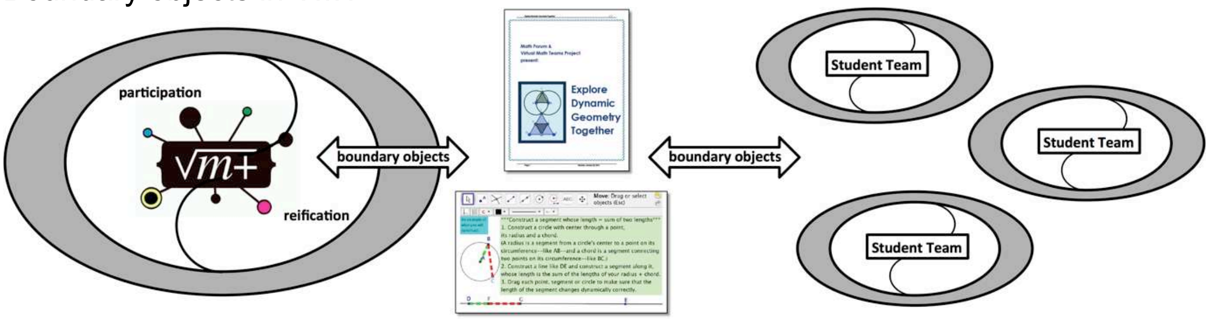

Figure 2. Boundary objects in the VMT project

From the point of view of CoP theory, participation in the VMT project team is built partly on community practices that support project members' knowledge of math, dynamic geometry, the project tools, and the team's educational and pedagogical philosophy. Reification in VMT takes this knowledge and produces artifacts such as the tool interfaces and documentation that represent this knowledge to users. Ongoing cycles of participation and reification have led to the creation of formalized webs of meaning that in turn support the team's practices and identity. This process has been going on for a number of years, with the team's knowledge of dynamic geometry reified in the form of a range of informational artifacts. Many of these artifacts are internal to the project, such as document drafts, meeting agendas and minutes, email, and so on. Other artifacts – boundary objects – have been designed to communicate the project team's understandings of dynamic geometry to wider audiences. Distillations of the team's thinking, these boundary objects are tailored for particular audiences, and include documents such as conference papers, journal articles, funding proposals, and reports. The boundary objects that we are interested in here are designed for students, and consist of sociotechnical ensembles of tools, interfaces, instructions, curricula, and other artifacts. As explicit reifications of the team's knowledge, they are designed to lead students, through a series of activities and exercises, to an understanding of dynamic geometry.

Figure 2 shows, on the left, the CoP of the project team; on the right, a series of small and emerging CoPs that represent various student teams; and in the middle, the boundary objects created by the project team and intended to serve as sense-making artifacts for the students. An important evaluation question here is: *Do these boundary objects – which make sense to the team – also make sense to the students and support them to learn about dynamic geometry?* To address this question, the students' discourse and actions in VMT are analyzed, using as a starting point Laborde's (2004) distinction between spatio-graphical and theoretical reasoning. According to Laborde, when students construct and use geometrical figures, they can understand these figures at 'face value,' in terms of what they see, and also in terms of wider theoretical reasoning. In learning, students oscillate between these two modes of reasoning, with theoretical hypotheses explored partly through the construction of diagrams, leading to new hypotheses being formed, and an increase in their overall understanding. From this perspective, when examining the VMT student logs, do we see students working their way through the assignments based on 'surface' comprehension of the visual forms that they see on their screens, or do they build theoretical arguments and reasoning to account for the underlying dynamic geometrical forms? The analysis that follows evaluates this question by examining the ways in which the project team and the students use the word 'construction' and its synonyms. Analyzing the use of 'construction' is useful, as such usage can be a marker of both spatio-graphical and theoretical reasoning, depending on the context in which it is used (c.f. Wittgenstein, 2001).

Methodological approach

As noted in the introduction, formative evaluation focuses on descriptions of ongoing dynamic processes, rather than static one-off measurements of project outcomes. It calls for different methods than may traditionally be used in summative evaluation. The research in this paper follows an ethnographic action-research approach (Tacchi, Slater, & Hearn, 2003), combining ethnographic methods with ongoing analyses of and contributions to the field site. The aim is to develop iterative improvements in theoretical and practical understanding, useful for both the ethnographer and the research subjects. It is a method suitable for complex field sites exhibiting organizational and technological development, such as VMT (Baskerville & Pries-Heje, 1999).

In this analysis, it is assumed that the VMT team produces reifications both for internal use and also for communicating with external stakeholders. The focus here is on boundary objects produced for students, and the students' responses to these boundary objects. The data examined were as follows. First, curricula and manuals related to the VMT tools were analyzed, including introductory assignments, and overall reviews of the project work (Stahl 2012, 2013b, 2014a, and 2014b). Second, the VMT interface, and instructions and assignments were analyzed (screenshots of many of these are included in the documents just cited). Third, the chat of a student group in the VMT Fall Fest 2013 was analyzed (see http://gerrystahl.net/vmt/icls2014/Topic3.xlsx; http://gerrystahl.net/vmt/icls2014/). The analysis followed a general grounded-discourse-analysis approach, in which 'construction' had previously been identified as a main category, and axial codes related to 'construction' were identified. The analysis was carried out using NVIVO coding software.

Results

As might be expected, the curricula, manuals, and the VMT interface, evidenced richer uses of 'construction' than the discourse of the student teams. At the same time, the students used what knowledge they had acquired from the reifications to engage, at times, in creative hypothesis generation. For reasons of space, the analysis in this section is abbreviated.

Curricula and manuals

Across the four documents, 'construction,' as well as 'constructing,' 'constructed,' and a range of synonyms, were used in a wide variety of practical and technical senses. Approximately 180 examples were identified. A common sense usage that emerged from the coding was that of an activity that can be carried out by users:

> GeoGebra lets you **construct** dynamic-mathematics figures

A related usage was that of a thing that was being made or had been made in GeoGebra by students or tutors:

> Take turns being in control of the **construction**. Say what you are doing in the chat.
>
> Use the chat to let people know when you want to 'take control' of the GeoGebra **construction**. Use the chat to tell people what you notice and what you are wondering about the **construction**.
>
> Sliding the history slider shows you previous versions of **constructions** in the GeoGebra tab, so you can review how your group did its work.

Construction was seen as an activity carried out with tools. For instance, the VMT tool has a 'construction area' (i.e. the whiteboard area and associated tool buttons):

> Here is how to use these tool buttons. Try each one out in the **construction** area of your own GeoGebra tab. First click on the button for the tool in the tool bar, then click in the **construction** area to use the tool.

Different uses of 'construction' were often combined in the same chunk of discourse, for example:

> You can even let someone else take control in your tab to help you **construct** something or to explore your construction. After your group **constructs** something in the group GeoGebra tab, you should make sure that you can do it yourself by doing the **construction** in your own tab.

A wide range of synonyms was also used to describe actions involved in construction, for example:

> Use the Compass to **draw** a circle whose radius is equal to the distance between two points and whose center is at a third point. First **click** on two points to **define** the length of the radius. Then without releasing the cursor, **drag** the circle to the point where you want its center to be.

A 'construction' was often seen as the goal or outcome of an assignment. This usage included subsidiary actions, such as creating, dragging, moving, and placing, and subsidiary components such as points, lines, segments, rays, and so on. While this whole/subsidiary distinction was often observed, there were also places where these usages overlapped, for instance where students were expected to construct a figure with underlying dependencies. In these cases, a dependency, although part of the overall figure, could be referred to in itself as a construction:

> Note: You must **construct** the dependencies among the objects, (lines and circle), not just **draw** something that looks like this.

Can you think of any ways you could use the dependency **created** with the compass tool or circle tool to **construct** other geometric figures or relationships?

Finally, where one task of the assignment was to come up with a component, this could also be referred to as a construction. For instance, a segment – if it was the outcome of an assignment – could be constructed:

Challenge: **Construct** a segment DG along ray DE, whose length is equal to the sum of the length of a radius AB of a circle plus the length of a segment BC connecting two points on the circumference of the circle.

Interface and assignments

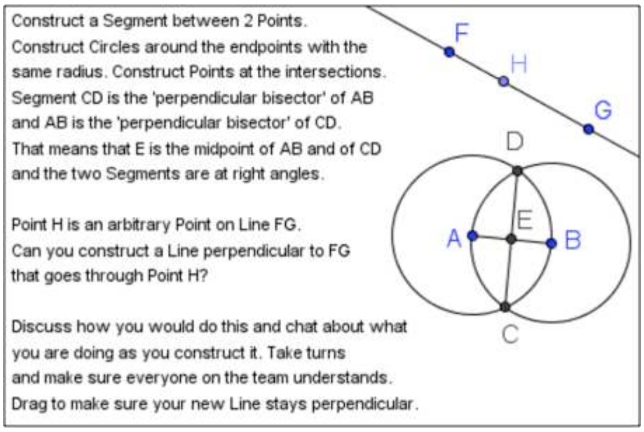

Figure 3. An example of an assignment

The project manuals (previous section) are useful documents, but students may not always read them, and their first encounter with VMT is often through the tool interface. The interface includes several interrelated functional elements, including a whiteboard area in which students can construct dynamic geometry figures with a range of tools, a chat window where they can communicate with other students in their group, and a list of users currently online. Often included in the whiteboard area are informational graphics related to the assignments. The content and format of the graphics varies, but they can be seen as summaries of the description of the assignment in the manual, consisting of a few sentences, step-by-step instructions, and figures (see Figure 3). Graphics are sometimes posted alongside pre-built Geogebra constructions that students can interact with. The assignments summarized in the graphics run from simple tasks that introduce the students to the functions and affordances of VMT, to more complex tasks that test students' understanding of dynamic geometry. As a boundary object, the assignment figure provides a good reflection and synopsis of the associated course materials, although it should be noted that as the description and accompanying figures are succinct summaries, they might require further background understanding in order to be usefully understood (such understanding as is, for instance, provided in the extended explanations in the accompanying manuals). The graphic illustrated in Figure 3 was presented to students in session 3 the Fall Fest of 2013, in which the students had to construct the perpendicular of a line they have previously constructed, and after that to construct a perpendicular bisector through a given point. The word 'construction' is used in several places in this assignment description, and refers both to the construction of various parts of assignment, and the overall goal of the assignment itself (the construction of perpendiculars and bisectors). The assignment also includes elements of dynamic geometry, where the students are instructed to 'drag to make sure your new Line stays perpendicular.'

Student chat logs and constructions

Students use VMT tools to discuss how to approach each assignment, to coordinate their use of the whiteboard and tools, and to discuss what they have learned and discovered in each session. The system is instrumented to capture the traces of student discussion and tool use in GeoGebra is. These logs are regularly analyzed by project members, both individually, and in collective data sessions. The discussion in this section refers to a student session associated with the GeoGebra assignment outlined in the previous section. An excerpt from this log is shown in Figure 4, which shows (left to right) the event number; the time a student started to write a comment; the time the student actually posted the comment; and the comment itself. Because of their login names, this student team, who are middle school students, are referred to by the VMT project as the 'Cereal Team' (sessions by the Cereal team are also reported in Çakir & Stahl, 2015, and Öner & Stahl, 2015).

Line	Start Time	Post Time	cornflakes	cheerios	fruitloops
28	37:27.9	37:40.4			so now you need to construck points at the intersection
29	38:02.8	38:12.1			no you dont make a line you make a line segment
30	38:31.3	38:35.1			good!!
31	39:17.4	39:20.4			so continue
32	38:26.4	39:29.9		i just made the intersecting line and point in the middle	
33	39:30.8	39:40.0		it made a perpindicular line	

Figure 4. An example of a VMT chat log

Here we can see the three team members, Cornflakes, Cheerios, and Fruitloops. An interesting feature of this session is that while the students often referred to using the tools, they used the term 'construction' relatively infrequently: they used 'make' fourteen times; 'put,' 'move' and other related terms seven times; and 'construct' only twice. For example:

(17) fruitloops: *how do i **make** the line segment?*

(29) fruitloops: *no you dont **make** a line you **make** a line segment*

(30) cheerios: *i just **made** the intersecting line and point in the middle*

(50) cheerios: ***turn** line fhg so its easier **make** it horizontal*

(57) cornflakes: *so after **construting** the line we **put** the circle on top*

(63) cornflakes: ***put** point m on tp of h*

(85) fruitloops: *you **make** the points go through qr and then you **move** h ontop of the line*

Where construction is part of the students' chat, it seems to be in terms of quoting from the assignment. For instance:

(28) fruitloops: *so now you need to **construck points at the intersection** (Assignment: **Construct Points at the intersections**)*

(53) cornflakes: *now **construct the line** (Assignment: Can you **construct a Line** perpendicular to FG that goes through Point H?)*

Line	Start Time	Post Time	cornflakes	cheerios	fruitloops
89	14:10.8	14:29.8			but how do we know for sure that the line is perpinmdicular
90	14:37.1	14:39.6		im not sure	
91	14:36.9	14:42.1	there 90 degree angles		
92	14:41.0	14:45.4		do u cornflakes	
93	14:46.1	14:59.4			but you cant really prove that by looking at it
94	14:55.4	15:06.8	they intersect throught the points that go through the circle		
95	15:02.1	15:17.7			it has to do with the perpendicular bisector
96	15:15.2	15:19.8	they"bisect" it		
97	15:20.0	15:31.2			and the circles
98	15:34.9	15:37.2		oh i see	

Figure 5. "You can't really prove that by looking at it."

One question that could be asked of the students' actions here, following Laborde's model of spatio-graphical and theoretical thinking, is: Are they generally making, moving, putting, etc., or are they 'constructing' in the dynamic-geometrical sense intended by the term? The discourse of the students describes various acts of manipulation, but there is little explicitly technical reference to any higher order dynamic-geometry principles that could be informing the construction. At the same time, however, the students also engage in some incipient attempts at proof, even if they lack the technical language to describe this in formal terms. This occurs towards the end of their session, where Fruitloops makes some suggestions for summing up the knowledge gained from the assignment (Figure 5). Here, Fruitloops seems to be working towards a distinction between what the students see, what they know, and what they should be able to prove, noting that "you can't really prove that [the line is perpendicular] by looking at it," but rather that the students should aim towards an understanding guided by deeper underlying geometrical thinking.

Discussion

One finding from the analysis is that the VMT project team used ‘construction’ and related terms in complex ways, while the students used the same terms imprecisely. On one level this should not be surprising. At this stage in their learning of dynamic geometry, the students do not necessarily have at their disposal the range of language and concepts that would enable them to make the rich connections between dynamic-geometry concepts and practices that the project team does. Further, the students seemed to respond literally to the assignment prompts, with the inference that they are demonstrating spatio-graphical responses; that is, they are talking about constructing without having full knowledge of what this might mean in dynamic geometry.

From the point of view of the evaluation framework proposed in this paper, one counter-argument is that the students did in fact develop an understanding of dynamic geometry, but that they did not (yet) possess a sophisticated enough vocabulary to express this in the same terms as the project team. Towards the end of the session described in this paper, and in response to the assignment instruction “Point H is an arbitrary Point on Line FG. Can you construct a Line perpendicular to FG that goes through H?”, they test hunches by moving the construction around to see if it aligned. They appear to be working towards a preliminary hypothesis regarding dynamic geometrical proof, even if they do not formally test this, or use the same terminology that a member of the VMT might have used. While from one perspective, this may be seen as spatio-graphical behavior, and lining up different figures with no regard for why they align, at the same time, these actions can also be seen as nascent forms of hypothesis testing based on early understandings of the possibilities for proof in dynamic-geometry environments. This has been suggested in other analyses of the Cereal Team. Çakir and Stahl (2015) studied another VMT session in which the team was instructed to drag previously constructed quadrilaterals, in order to make inferences regarding the underlying dependencies and constructions of these quadrilaterals. They note: “through an interactive process of calibrating and recalibrating their indexical references (Zemel & Koschmann, 2013) to the evolving visual configurations witnessed during different dragging performances, the team members were able to collectively notice several dependencies among constituent elements, describe them in colloquial and semi-formal terms, and produce conjectures for the underlying causes of those dependencies.” Similarly, Öner and Stahl (2015), who have analyzed the same session described in this paper, but using Sfard’s commognitive framework, suggest that by the end of the session the students are engaged in working towards a preliminary discussion of proof, even if this is not framed in rigorous terms.

From the perspective of Laborde, the Cereal Team iterated rapidly between spatio-graphical responses and theoretical reasoning. This rapid iteration and reasoning was supported by the dynamic nature of the VMT tools and the ability to drag, as well as by the collaborative affordances of the interface, such as the ability to watch other students manipulating constructions in real time, and the chat window for discussion of these manipulations. This iteration was productive; while (as the students observed) the perpendicular bisector figure may look the same from either a spatio-graphical or theoretical perspective, as they interacted with it over the course of the session, the students started to specify differences between these perspectives. They began to posit proto-rules for evaluating whatever it is that they have been instructed to notice in the assignment (constructing a perpendicular bisector), and move from spatio-graphical reasoning and towards theoretical reasoning. Thus, while their discourse consciously reflects at least partly the steps presented in the assignment images and texts – and they recognize that following these steps correctly should result in achieving the assignment goal – they are also aware that the assignment calls on them to provide theoretical accounts beyond those same steps. They recognize that their dynamic constructions can be seen both as spatio-graphical assemblages of components (points, lines, circles), and also as objects that can be explained in terms of more abstract underlying dynamic-geometrical principles. At this stage, however, this reasoning is emergent, not least because of the lack of practice with and reflection upon dynamic geometry, at least beyond the immediate goals of the assignment.

Overall, the VMT tools and documentation functioned well as boundary objects that reified and externalized the project team’s knowledge for students. This is not surprising, given the design research approach that the project team uses, which produces regular ongoing revisions to these artifacts. At the same time, the observations, framed within CoP theory, regarding the differences in meaning of ‘construction’ in both the vocabularies and practices of the project team and the students, suggest further topics for investigation in this process. What are the ontological grounds that the team and the students bring to their understanding of geometry and dynamic geometry? Do the students understand their actions in terms of classical geometry, in which they have yet to be formally trained, or in terms of a translation of dynamic geometry, or in terms of something else again? Further, how do they interpret the affordances of the VMT tools? Do they assume that objects move, unless otherwise specified, and if so, what sort of understanding of dynamic geometry then emerges? These questions lead to a consideration of how VMT can be developed further to support students in theory building and proof. One strategy suggested by the evaluation is to continue to refine the boundary objects produced by the team (in the form, for instance, of assignment images and texts), and to gain further traction with the students' understanding of what is meant by 'construction.' A second strategy is to understand further the contribution the connections between the collaborative nature of the tools, and the rapid iterations between spatio-graphical and theoretical thinking displayed by the students. (Note: this section benefited from discussion at the 2014 ICLS workshop *Interaction Analysis of Student Teams Enacting the Practices of Collaborative Dynamic Geometry*; http://gerrystahl.net/vmt/icls2014/.)

Conclusion

A formative evaluation of the VMT project, based on Communities of Practice and boundary objects, identified various uses of 'constructing' by project members and students. For the VMT team, the idea of constructing was constituted within a web of dynamic geometry knowledge, and reified in boundary objects such as instructional manuals, assignments, and interfaces. The students drew on these boundary objects, developing notions of 'constructing' which were more emergent. The evaluation recommendations are that the project's boundary objects should continue to be refined, and also that further understanding be gained of what team members and students understand by 'constructing' and related terms. Overall, the CoP approach usefully pulled back the evaluation lens from the tools, and brought into view the project as a whole, covering not just technology use, but organizational levels of design and implementation. The evaluation design allowed insights to be fed back to the project team on an iterative basis, complementing the design research approach of the team.

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