The Function of Epistemic Emotions for Complex Reasoning in Mathematics

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Abstract: This study examined how epistemic emotions, motivational mechanisms, and their effects are linked during complex reasoning in mathematics. Data were collected from 80 university students enrolled in mathematics study programs. Participants first answered a series of multiple choice questions regarding knowledge of basic geometry theorems and afterwards solved a complex proof task. Immediately upon task completion, students reported their epistemic emotions as well as motivation during the task. Path analyses revealed that epistemic emotions during problem solving predicted the motivation students invested in solving the complex reasoning task. Finally, intrinsic motivation positively predicted task performance. The findings suggest complex linkages of epistemic emotions with motivational mechanisms during reasoning and knowledge generating processes and demonstrate the adaptive and maladaptive functions of epistemic emotions. Consequently their role for complex reasoning should be considered in future research and can guide instructional design to foster reasoning in mathematics.

Introduction

In mathematics, deductive reasoning is one of the fundamental modes in order to generate evidence for or against a claim or theory, marking mathematics oftentimes as a "cold" science (Sinatra, Broughton, & Lombardi, 2014). However, in order to solve more complex problems and advance scientific discovery in mathematics, reasoning paths have to be refuted, approaches to solutions have to alter between heuristic and analytical strategies, and complex or even contradictory information must be evaluated and at times reconciled. From this perspective, mathematical reasoning is prone to be affected by emotional mechanisms driving epistemic processes (Fischer et al., 2014; Muis et al., 2015). Systematic research examining those links is largely lacking. To address this research deficit, we are investigating the role of epistemic emotions for complex scientific reasoning in the domain of mathematics.

Epistemic emotions have been described to relate to the cognitive quality of tasks and information processing (Pekrun & Stephens, 2012), and consequently their object focus is on knowledge and processes of knowing (Muis et al., 2015). The influence of epistemic emotions on performance and achievement can be theoretically explained by their impact on various variables, including the motivation invested in a learning or problem solving activity (Pekrun, Goetz, Titz, & Perry, 2002; Pekrun & Perry, 2014). The theoretical framework for epistemic emotions proposes that epistemic emotions can positively affect knowledge-generating activities and motivational processes during task engagement. In particular, both positive activating (e.g., curiosity) and negative activating epistemic emotions (e.g., confusion) have been associated with beneficial effects for learning (e.g., D'Mello, Lehman, Pekrun, & Graesser, 2014). By contrast, the negative deactivating emotion boredom has been found to be detrimental (Tze, Daniels, & Klassen, 2015). However, as compared to cognitive effects (Muis et al., 2015) the role of epistemic emotions for motivational mechanisms during complex reasoning has rarely been studied. Consequently, we are investigating if and how epistemic emotions effect complex reasoning in mathematics through motivational mechanisms.

We base our exemplary hypotheses on those few empirical findings which have found positive activating epistemic emotions, such as epistemic curiosity or enjoyment, to consistently motivate exploration (Litman et al., 2005) as well as students engagement (Pekrun et al., 2002) but to also promote intrinsic motivation (Pekrun & Stephens, 2012) during complex reasoning. Boredom, a negative deactivating epistemic emotion, has been associated with task disengagement (Tze, Klassen, & Daniels, 2014; D´Mello & Graesser, 2012) and the lowering of intrinsic motivation (Pekrun et al., 2010) and is therefore expected to reduce the intrinsic motivation to solve the task while it is also expected to have a positive relationship with amotivation. Assuming mediating functions of motivational variables between epistemic emotions and task performance (Pekrun, 2006), we hypothesize that, in turn, intrinsic motivation should be associated with positive outcomes and consequently will positively predict task performance while, for example amotivation, should lead to withdrawal from the task and would consequently have a negative effect on task performance.

Method

Sample

A total of 80 University students from a German University (M_{Age} = 22.91, SD_{Age} = 4.54; 45 male (56.3%), 33 female (41.3%), 2 missing) participated in the study. All students were enrolled in study programs of mathematics ($M_{Semester}$ = 4.28, $SD_{Semester}$ = 3.07).

Materials and measures

Mathematical performance

A geometric proof task was developed based on psychological models of knowledge and deductive reasoning (Ufer, Heinze, Reiss, 2009; 2009a). Here, a geometric stimulus has to be investigated by the participant by applying knowledge of mathematical theorems in order to generate a cohesive proof. A coding scheme was developed and based on two independent raters their concordance was assessed with Cohen's Kappa [*k*] which ranged from 0.74 to 0.96 [*M* = .83, *SD* = .11] indicating overall good reliability. Based on these rated variables the following measures were agglomerated: 1) the number of arguments logically constructed to formulate the proof (based on premises and conclusions); 2) the number of arguments to which a mathematical justification had been added (i.e., a direct reference to a mathematical theory or axiom); 3) the quality of those arguments. As normal distributions were different for each of the three performance measures, z-scores were computed in order to form an overall performance measure for the constructed proof (1).

Prior knowledge

Prior knowledge in Geometry was assessed based on a series of multiple choice questions regarding knowledge of basic geometry theorems before students solved the proof task. One point for each correct answer was given, which resulted in a range of possible scores from 0 to 35 (*M* = 30.43, *SD* = 3.72, min. = 15, max. = 35).

Epistemic emotions

Epistemic Emotions that students experienced while solving the complex proof task were measured using the short version of the Epistemic Emotions Scale (EES, Pekrun & Meier, 2011). Each item consisted of a single word describing one emotion (e.g., curious, surprised, confused, anxious, frustrated, excited, bored) and were assessed as intensity ratings (1 = *not at all* to 5 = *very strong*).

Motivational mechanisms

A multidimensional measure of the contextual motivation to perform the task was employed based on the Situational Motivation Scale (SIMS; Guay, Vallerand, & Blanchard, 2000). Each of the following constructs based on four items was assessed: intrinsic motivation, identified motivation, external motivation and amotivation. The intrinsic motivation scale assessed the degree to which a participant reported to having solved the task in order to experience pleasure or satisfaction inherent in the activity (e.g., "Because I think that this activity is interesting"). In contrast, the extrinsic motivation scale measured compliance beyond the activity itself (e.g., "Because it is something that I have to do"). The identified motivation subscale referred to the personal importance or conscious value individuals experienced when they were engaged in the activity (e.g., "Because I am doing it for my own good"). Last, the amotivation scale measured to what extent participants saw no sense of purpose or value in solving the geometry task (e.g., "There may be good reasons to do this activity, but personally I don´t see any"). All self-report scales were adjusted to fit the task specific context of geometry (Instruction: "Why [were] you engaged in this [task]?") and collected via self-report on 5-point Likert Scales (1 = *not at all true of me* to 5 = *very true of me*) so that a higher score on each scale represents a stronger endorsement of the corresponding construct.

Analysis

Preliminary analyses

Pearson product-moment correlations were calculated for all study variables. The epistemic emotions were related in the expected directions, for example, positive activating emotions (curiosity and excitement) were positively correlated while boredom, a negative deactivating emotion, showed a strong negative relationship to curiosity as well as excitement. Further, confusion was substantially related to frustration but both negative activating epistemic emotions were unrelated to anxiety. Additionally, confusion was negatively related to excitement and surprise.

Curiosity, for example, was systematically related to all motivational variables and showed a strong positive relationship with intrinsic as well as identified motivation while the relationship with external motivation and amotivation was strongly negative. A similarly consistent pattern emerged for boredom. While this deactivating epistemic emotion was negatively related to intrinsic and identified motivation a strong positive relationships was found for amotivation.

Last, only excitement, frustration and boredom were related to task performance. Prior knowledge was significantly related to task performance and consequently included as a covariate in the following analyses.

Path analyses

Path analyses were conducted to test the mediation model. Hayes and Preacher´s (2013) MEDIATE SPSS macro was used with manifest variables to account for small sample size. To test significance, the macro uses bootstrapping (generating a sampling distribution of the effects by pretending the sample is a population and drawing random 10,000 resamples). The Monte Carlo method (Preacher & Selig, 2012) is here applied to estimate paths coefficients. Prior knowledge was included as a covariate for all variables in the model. Mediation analysis tested, while including all variables in one step, whether motivational variables mediated relations between epistemic emotions and task performance.

The analysis included all epistemic emotions as predictors, all motivational variables as mediators, performance as the outcome variable, and prior knowledge as a covariate. The total effects model was significant. Curiosity and excitement positively predicted intrinsic motivation. Identified motivation was positively predicted by curiosity and negatively predicted by boredom. Although the model for predicting extrinsic motivation reached significance only excitement approached significance as a negative predictor. Last, boredom was a positive and curiosity a negative predictor for amotivation during the completion of the task. The model that included task performance as an outcome variable was significant and was positively predicted by intrinsic motivation and prior knowledge.

Lastly, indirect effects revealed that intrinsic motivation experienced during the task mediated the relations between curiosity as well as excitement during problem solving and task performance.

Conclusions and implications

In order to better understand student reasoning and learning during complex reasoning in mathematics, it is crucial to carefully untangle all components underlying and determining student performance. This study examined the role of epistemic emotions experienced during a complex mathematical task in predicting motivational mechanisms and consequently task performance. Results revealed that epistemic emotions predicted different motivational variables during task performance, while in turn, intrinsic motivation positively predicted task performance. Specifically, our findings suggest that positive activating epistemic emotions act as important drivers for perseverance during complex reasoning problems and can profoundly impact the quality and outcomes of cognitive processes (Fischer et al., 2014, Pekrun, 2006). On the other hand, our findings additionally indicate that deactivating emotions may help to explain withdrawal from more complex reasoning tasks. In sum, these complex linkages of epistemic emotions with motivational mechanisms demonstrate the importance for understanding the function of epistemic emotions for reasoning and knowledge generating processes and make their adaptive and maladaptive effects salient. The findings highlight the need to consider epistemic emotions alongside motivational variables when conducting research on reasoning and designing classroom instruction.

Endnotes

(1) All three z-scores were added and then divided by the total number of measures (3) to form an overall indicator of task performance.

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