Examining the Dual Function of Computational Technology on the Conception of Mathematical Proof

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Abstract: In this paper, I examine how the availability of a certain technology and new ideas about the nature of learning operate as a factor to suggest a novel understanding about a crucial mathematical concept: mathematical proof. I characterize the resulting conception for ideal mathematical proof activity combining two fundamentally different ways of knowing: *a posteriori* (or experimental/empirical) and *a priori* (or deductive/propositional). Obviously, such conception of proving is/will be central in designing proof tasks, thus shaping the mathematical discourse around proof within classrooms. Whether one considers participation within such discourse is simply an aid or tantamount to thinking, identifying the character of this discourse appears to be essential in order to examine the interrelationships between what is social and what is individual.

Salomon (1998) argues that technology serves a dual function resulting with a reciprocal relationship between technology and our understanding of human learning. On the one hand, the use (and the design) of technologies are informed and guided by the theories of learning. On the other hand, affordances provided by new technologies offer novel learning experiences, which in turn compels us to reconsider our conceptions about learning. In this paper, I aim to draw attention to yet another mechanism between technology, society, and human mind within the context of mathematical proof and new computer tools.

Traditionally, proving activity has often been conceptualized along one dimension: Providing a *deductive* argument. That is, students are either presented or asked to write rigorous logical arguments to establish the truth of the mathematical theorems. This represents *a priori* (Kant, trans. 1998) way of knowing, since one reaches a *true* conclusion by starting with a set of axioms, the building blocks of any mathematical structure, and applying the rules of logic to those axioms. Empirical or *a posteriori* (ibid.) ways of knowings are almost always left out within proving activity. I argue that the interaction between two major forces challenges such conception of proving.

One of the forces to challenge the conception of students' proving activity as a merely deductive experience is the recent mathematics education reform movement led by the NCTM (1989, 2000). In NCTM documents, "'[k]nowing' mathematics is 'doing' mathematics. A person gathers, discovers, or creates knowledge in the course of some activity having a purpose" (NCTM, 1989, p. 7). This process and meaning-centered perspective has led researchers to think about what makes proof *meaningful* and thus to reexamine the nature of mathematicians' proving activity.

As a result, there is a growing emphasis on the role of empirical explorations in mathematics education (Hoyles, 1997). While earlier conceptions of proof excluded empirical ways of knowing within proving activity, new ideas about the nature of learning have proposed the opposite. Researchers have come to believe that exploration/experimentation of mathematical ideas is an important ingredient of proving, for it reflects the work of expert mathematicians, gives students the opportunity to work from their own intuitions and investigations, and thus potentially makes proving more meaningful and accessible (Boero, 1999; Edwards, 1997; Reiss & Renkl, 2002).

The realization of the aforementioned vision of proving has been enabled by the immediate availability of a very powerful set of computer tools, namely dynamic geometry software (DGS), in classrooms. DGS materializes the vision described above, due to its most defining feature: dragging. That is, when the elements of a drawing are moved, this feature allows the construction to respond dynamically to the altered conditions (Goldenberg & Cuoco, 1998) by maintaining the invariant. This aspect of DGS facilitates conjecturing and more inductive approaches to geometric knowledge, as students can reason about the generality of their hypotheses for several cases (Kaput, 1992).

However, as much as DGS attracts great interest, concerns have been raised that students using DGS could be misconceptualizing the nature of mathematical truth; that is, coming to believe that a confirmation of a conjecture for several cases would secure its truth (Allen, 1996; Chazan, 1993b). As conviction can be obtained easily by dragging, DGS environments may prevent students from understanding the need and function of proof (Hadas, Hershkowitz, & Schwarz, 2000).

Despite these concerns, however, other researchers have maintained their position arguing that there is no tension between proof and empirical exploration (de Villiers, 1997, 1998; Hoyles & Jones, 1998). Researchers view working with DGS as an opportunity to emphasize the *explanation* function of proof (that is, providing insight into why the theorem is true) as a viable alternative to the notion of proof whose function has been chiefly verification, as in the traditional teaching approach (de Villiers, 2003; Hanna, 2000). Further attempts also have been made to show with empirical studies that DGS could be a useful tool to teach proof (Hadas et al., 2000; Healy & Hoyles, 2001; Jones, 2000; Mariotti, 2000, 2001; Marrades & Gutierrez, 2000).

This interaction between the ideas of reform and the immediate availability of computational technology in classrooms leads us to a notion of proof that encompasses both empirical and deductive ways of knowing. Moreover, they are considered complementary and reinforcing each other. However this does not suggest that exploration and formal proof are separate activities and mediated by different tools. Research shows that computational technology also problematizes the discrete positioning of the two different ways of knowing. In other words, there is evidence to think against the idea that one can only make exploration phase efficient with DGS and students turn to paper and pencil in order to give deductive arguments for their conjectures.

In mathematics education research, there is a growing recognition of the "reorganizer" (Pea, 1985) nature of DGS. That is, several authors point out that DGS is not simply making the task more efficient, rather it fundamentally changes the task (Healy & Hoyles, 2001; Jones, 2000; Lerman, 2001). Within the DGS learning environment, "[t]he computer is more than a mediating bridge, as its function cannot be simply reduced to a learning aid – to be discarded after the concepts and procedures have been acquired" (Holzl, 2001, p. 81). According to Scher (1999), DGS is also exerting its influence on the proving activity. He criticizes the body of work that limits DGS use to only explorations. For him, the boundary between deductive reasoning and dynamic geometry becomes increasingly blurred.

Referencing Salomon (1998), in this paper, my purpose was to highlight how the dialectic between two forces has resulted with a new conception for mathematical proof. It appears that the way educators understand and think about proof has a new meaning, resulting from a mutual relationship between the ideas of reform and the availability of computational technology. Both deductive/propositional and empirical/experimental ways of knowings are equally incorporated in this revised conception. Moreover, computational technology also compels us to rethink about the distinction between exploration and deductive proof. This conception does and will shape the mathematical *discourse* within which students are expected to participate. And the emerging notion of mathematical proof discussed in this paper proposes a layer of understanding regarding the mathematical discourse around proof.

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