From Visualization to Logical Necessity, Through Argumentative Design

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Abstract: We show that a meticulous design can encourage students in dyads to shift from informal reasoning (visual, inquiry-based) to reasoning moved by logical necessity (abductive and deductive). We describe a case study in which one dyad solves a series activities purposely designed. We show that argumentation first relies on intuition, and then intertwines the activities of conjecturing and checking the conjectures though the use of different gestures.

With the multiplicity of tools available in mathematics education, instructional design can do miracles: in this study, we show that one can encourage students to shift from informal reasoning to reasoning moved by logical necessity. We show how various kinds of reasoning processes (visual, inquiry-based, abductive and deductive) can stem from interactive argumentation. The activity of proving which was always difficult to trigger, can stem from argumentation. The use of special software helps to prop on intuitions, and on visualization. In a proper environment, dyads use gestures that accompany verbal argumentation. Productivity of collective argumentation consisted not only in a shift in reasoning type (towards deductive considerations), but in co-construction of knowledge – in the progressive emergence of geometrical principles combined in arguments. These positive outcomes resulted from collaborative rather than adversarial interactions as the students tried to accommodate their divergent views.

The research design experiment

Leading researchers have recognized the importance of CSCL tools in learning mathematics (e.g., Stahl, 2009). Researchers in mathematics education have invested a special effort in designing situations incorporating computerized tools for learning (e.g., Hadas, Hershkowitz & Schwarz, 2001, 2002). In the present study, (1) we design an activity in geometry as an inquiry-based activity that invites participants to raise and check hypotheses and encourages them to engage in argumentation which is productive in the sense that it leads to deductive reasoning; (2) we identify and analyze the various kinds of reasoning processes students adopt (visual, inquiry-based, deductive) while they are engaged in such an activity and show how these ways of reasoning are interwoven in collective argumentation; (3) we trace dynamic changes and development of individuals' and collective's argumentation processes of peers of students, working on such a designed activity - from what is usually called: "informal" kinds of reasoning to more "formal" ways. (Rasmussen et al, 2005).

Research subjects

The subjects in this research are pre-service teachers trained to teach mathematics at the elementary school level. They participate in a course about "Geometry and Computerized Environments", in a Teacher's college in Jerusalem. The pre-service teachers (called 'students' onward) generally worked in dyads. They were familiar with Dynamic Geometry software tools.

Analysis of data

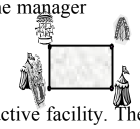
We adopted a methodological approach used by Rasmussen and Stephan (2008), who used Toulmin model of argumentation for tracing and analyzing the argumentation process. We analyzed with this method the protocol of each dyad in each task. We identified the characteristics of the arguments the dyad developed, including the resources and reasons (warrants and backings) invoked. We looked at the kinds of interactions that developed; also we discerned shifts in the argument schemes in the collective argumentation within or between tasks.

To achieve the research goals, we had first (as we mentioned before) to design the activity, its environment and conditions, in such a way as to encourage students to engage in argumentative processes that would lead to the elaboration of logic deductive argumentation

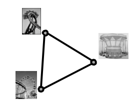
The designed activity

Figure 1 shows the activity we designed. It was designed according to three design principles: (1) creating a conflict situation (2) creating a collaborative situation (3) providing an environment for raising and checking hypotheses. It is organized in three successive tasks. In each of the three tasks, students were asked to go through similar phases: i) to find individually a solution for the shape that was drawn on the worksheet. In this phase they could check their solution with a first hypothesis testing device, a ruler for measurement ii) to work in dyads and to reach a consensus concerning the solution (collaboration principle) iii) to check their conjectured solution. At this phase, the hypothesis testing device was the Dynamic Geometry software with which they could undertake manipulations (hypothesis testing/checking principle). In case they made a wrong conjecture, they were asked to make a new conjecture and recheck it with the DG tools.

Task 1:
A recreational park has a rectangular shape. At each vertex of the rectangular park there is an attraction. The manager of the park decided to locate the ticket booth at an equal distance from the four vertices of the rectangle.
Find the point in the rectangle, which fits for the ticket booth



Task 2:
Another park has the shape of an equilateral triangle. At each vertex of this triangular park there is an attractive facility. The manager decided to locate the tickets booth at an equal distance from the three vertices of the triangle.
Find the point in the triangle that would satisfy the requirements.



Task 3:
What if the recreational park has the shape of a scalene triangle?
Find the point (if any) in this triangle that would satisfy the requirements.

Figure 1: The sequence of tasks (A shortened version)

The purpose of Task 1 is to get acquainted with the issue of finding a point which is equidistant from the vertices of a given figure. In Task 2, the shape of the recreational park is an equilateral triangle. Our assumption was that students would choose "special lines" – the three medians, the angle bisectors, or altitudes, and would find the intersection point of the three special lines, as a solution for the task.
Our assumption was that the students would imagine that the methods they used in Task 2 would be suitable for Task 3 as well, meaning that the solution would be the intersection point of the medians, or of the angles' bisectors, or of the altitudes. None of the above is the right solution, so students enter a conflict situation. Intuitions could certainly help in Task 1 and probably in Task 2, but naturally led to a wrong conjecture in Task 3 (conflict principle). In cases that they faced conflict - their conjecture was wrong; they were "pushed" to propose a new conjecture and to justify it.

Analysis and Implications

The activity we will analyze, invited the use of concrete methods and visualization in argumentation, but at the same time offered opportunities that point out the limitations in adopting such methods. We will present a case-study, in which students realize the importance of using Dynamic Geometry software for checking their conjectures, and yet find themselves in a dead-end position (conflict situation) and face the need to adopt logical-geometrical reasoning to find and justify the solution of their problem.

All dyads that coped with the activity were led to believe that the methods they used in the second task would be suitable for the third task as well, so the designed conflict situation was achieved. On the poster we will focus on the story of one dyad. We will demonstrate through this story how argumentation developed from being based on intuition and visual considerations to deductive considerations. The apparatus in which argumentation develops – different tools at disposal and the presence of two collaborating students, affords shifts in argumentation, not only in the person leading it, not only in its structure (focused on reasons instead of claims), but in the nature of arguments evoked, as reasons (warrants, backing) progressively become deductive. In this transformation, multimodality plays a central role. We will show that visual elements can serve as precursors of arguments, but sometime they are not enough and deductive considerations are the only solution. We argue that beyond the general lessons that we will present concerning the importance of design principles for learning in dyadic interaction, the specific achievement of this study, is the fact that the outcome of the dyadic interaction was deductive proving as an activity that convinces its actors.

Selected references

Andriessen, J. E. B. & Schwarz, B. B. (2009). Argumentative design. In N. Muller-Mirza and A. N. Perret-Clermont (Eds.), *Argumentation and Education – Theoretical Foundations and Practices* (pp. 145-174). Springer Verlag.

Hadas, N., Hershkowitz, R., & Schwarz, B. B. (2001). The role of surprise and uncertainty in promoting the need to prove in computerized environment. *Educational Studies in Mathematics, 44*(1&2), 127-150.

Rasmussen, C. & Stephan, M. (2008). A methodology for documenting collective activity. In A.E. Kelly, R. A. Lesh & J. Y. Baek (Eds.), *Handbook of design research methods in education* (pp. 195-215). New York and London: Routledge Publishers..

Stahl, G. (2009). Mathematical discourse as group cognition. In G. Stahl (Ed.), *Studying Virtual Math Teams* (ch. 3, pp. 31-40). New York, NY: Springer.