New Evidence on Productive Failure
- Building on Students’ Prior Knowledge is Key!

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Abstract: The work on Productive Failure (Kapur, 2009) suggests that students can benefit from generating representations during collaborative problem-solving followed by instruction that builds on the student-generated representations. However, Productive Failure has usually been compared to Direct Instruction in studies with several confounds. These evaluations, therefore, do not allow conclusions regarding the underlying learning mechanisms. To address this criticism, we compared four conditions in a quasi-experimental study with 154 10th graders: We varied the form of instruction in two Direct Instruction conditions (a regular DI condition and a DI-S condition where instruction built on typical student-generated representations) and the type of support students received in two Productive Failure conditions (a regular PF condition and a PF+ condition with additional cognitive support). Our results indicate that students benefit from introducing new concepts based on student-generated representations. Moreover, during generation motivational support seems sufficient, that is, students do not need cognitive support.

Collaborative Learning with a Diversity of Representations in Mathematics

The mathematics education literature points at the importance of supporting students in developing an understanding of different representational formats, such as tables, graphs, and algebraic notations (e.g. Cuoco & Curcio, 2001). Teaching students to understand and connect different representational formats not only improves their representational competency, but also fosters their understanding of the underlying mathematical concepts and problem-solving procedures. These assumptions are in line with findings from educational psychology research concerning the added value of multiple representations for learning (e.g. Ainsworth, 2006). However, several studies also demonstrated that students do not automatically benefit from multiple representations. One problem that has been identified is that students often tend to treat the representations in isolation instead of making connections and integrating the information provided by the different representations (e.g. Ainsworth, 1999, 2006). One possible approach to promoting learning with multiple representations, and particularly to promoting connection-making across representations, might be to include collaborative learning opportunities (Bodemer, Kapur, Molinari, Rummel, & Weinberger, 2011; Rummel & Braun, 2009): Within a small group, members may have different perspectives on the representations. By discussing and elaborating the different perspectives, a deeper processing of the mathematical concepts underlying the representations may be fostered as students co-construct a shared understanding that goes beyond the understanding of each individual (Moschkovich, 1996). Indeed, research on collaborative learning shows that students discuss connections between information provided by different representations (e.g. Suthers & Hundhausen, 2003). Similar to the learning environment used in the study by Suthers and Hundhausen, many collaborative learning environments (especially computer-based environments) include different external representations; however, research has usually focused on how to support collaboration by introducing representations, and rarely the other way around. Hence, it remains to be investigated how to best design collaborative settings in order to support learning with a diversity of representations.

Based on the way mathematics textbooks and courses are most commonly designed, collaborative learning with different representational formats would likely be implemented in the following way: The textbook or the teacher introduces a new mathematical concept by using different representational formats, for example, symbolic representations (i.e. formulas) and graphical representations (e.g. diagrams). Afterwards students practice problem-solving using these representational formats in small groups. An alternative approach would be to begin by having small groups of students interact with different representations based on their formal and informal prior knowledge, and have the teacher provide instruction thereafter. In other words, students first generate and/or learn with different representations in order to develop an intuitive understanding of the targeted mathematical concept and only afterwards the teacher provides instruction building on these representations. Such a delay of instruction lies at the heart of the Invention Paradigm (e.g. Kapur, 2009; Roll, Aleven, & Koedinger, 2011; Schwartz & Martin, 2004). One approach within the Invention paradigm is Productive Failure (e.g. Kapur, 2009). In a series of experiments conducted by Kapur in Singaporean classrooms, students in a Productive Failure condition (PF) engaged in problem-solving activities in small groups without receiving instructional support. Process data showed that students collaboratively generated a diversity of representations during this phase. Thereafter, typical student-generated representations were
contrasted in a teacher-led discussion which finally led to the presentation of the canonical solution by the teacher. Although students in the PF condition were rarely able to find the canonical solution during collaborative problem-solving, they significantly outperformed students who first received Direct Instruction (DI) in the posttest (e.g. Kapur, 2009). Upon closer inspection, however, students in the DI condition in fact received a different form of instruction than students in the PF condition: The teacher directly presented the canonical solution, rather than building on typical student-generated representations and misconceptions. Thus, when comparing the two conditions, the *timing of the instruction* and the *form of instruction* were confounded.

It could be hypothesized that the DI setting would also benefit from a classroom discussion about typical student-generated representations and misconceptions (Hammann, 2003). Furthermore, in the PF condition it is so far unclear whether a complete lack of support during collaborative problem-solving is indeed optimal for learning. Advocates of the Invention Paradigm seem to argue for a delay of instruction and support. In contrast, the broader literature on collaborative learning suggests supporting students’ collaborative activities at least to some extent as fruitful collaboration does not occur automatically (O’Donnell, 1999). It can be assumed that in the context of the Invention Paradigm, the benefits of collaborative activities would also unfold best if some support was given (Westermann & Rummel, 2012). It therefore seems to be a logical next step to investigate what type of support students need when they collaboratively learn with a diversity of representations. Indeed, in the Productive Failure studies students received at least some support during problem-solving: In earlier studies on Productive Failure (Kapur, 2009), students received motivational support to persist in generating different representations. In later studies by Kapur (personal communication, March 2010), students additionally received cognitive prompts in form of contrasting cases to their generated solution approaches. These prompts should support students to improve their solution approaches from one representation to the next. However, the differences between the *support features* in the former and the latter studies have not been investigated empirically so far.

**Methods**

In our study, we aimed to shed light on the impact of building instruction on typical student-generated representations and misconceptions, and on support features that might make the problem-solving phase of Productive Failure even more effective for learning. In order to address these questions, we compared four conditions in a quasi-experimental study as shown in Table 1: We varied the form of instruction in two Direct Instruction conditions (a regular DI condition and a DI-S condition where instruction built on typical student-generated representations) and the type of support students received when collaboratively generating representations in two Productive Failure conditions (a regular PF condition and a PF+ condition with additional cognitive support).

Table 1: Conditions of the study.

<table>
<thead>
<tr>
<th>Learning phase 1</th>
<th>Learning phase 2</th>
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<tbody>
<tr>
<td><strong>DI</strong></td>
<td>Instruction with canonical representations</td>
</tr>
<tr>
<td><strong>DI-S</strong></td>
<td>Instruction based on typical student-generated representations</td>
</tr>
<tr>
<td><strong>PF</strong></td>
<td>Problem-solving in small groups</td>
</tr>
<tr>
<td><strong>PF+</strong></td>
<td>Problem-solving in small groups with cognitive prompts</td>
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The regular *Direct Instruction condition* (DI) served as a control condition. In the DI condition the teacher explained a yet unknown concept (the concept of variance and deviation) and introduced the canonical solution by using different representational formats. In order to address the confound between timing of the instruction and form of the instruction, we implemented a second *Direct Instruction condition* (DI-S) where the teacher introduced the canonical solution by discussing and building on typical student-generated representations (i.e. representations that students in the Productive Failure conditions typically generate during problem-solving). In both Direct Instruction conditions the instructional phase was followed by problem-solving in small groups of three students.

Both Productive Failure conditions started with problem-solving in groups of three students to the yet unknown concept. Students were instructed to try different solution approaches by generating different representations, such as tables, graphs and formulas. They used tablet PCs to generate representations. During this phase students in the standard *Productive Failure condition* (PF) only received motivational prompts encouraging them to persist in solving the task (e.g. “you are doing a good job together, keep going”). In the *augmented Productive Failure condition* (PF+), students additionally received cognitive prompts, that is, students were supported in their critical evaluation of the representations they were generating by providing
them with contrasting cases to their solution approaches (e.g. “maybe there are situations where your solution
does not work, have a look at this (counter-)example”). In both Productive Failure conditions the problem-
solving phase was followed by an instruction phase where the teacher introduced the canonical solution by
building on typical student-generated representations as in the instruction phase of the DI-S condition.

Participants were 154 10th graders recruited from two secondary schools in Germany. Learning
outcomes were assessed by a posttest after the second learning phase. The posttest included items testing for
procedural skills and for conceptual knowledge. We further recorded process data (video and screen recordings)
of students’ interaction during the collaborative problem-solving phase. This data enables us to investigate the
possible impact of the cognitive prompts in the PF+ condition on the collaborative learning process. The
analysis focuses on the number and quality of the generated representations as well as on how students
 collaboratively generate and learn with multiple representations.

Results

Only the 144 students who were present during both learning phases were included in the analyses. A
factor analysis of the posttest items revealed two factors with an eigenvalue of 1 or more (2.47, 1.26), that
accounted for 53.53% of the total variance. The rotated factor loadings were in line with our intended
differentiation between procedural skills and conceptual knowledge. The items are shown in Table 2.

Table 2: Rotated factor loadings.

<table>
<thead>
<tr>
<th>Item 1: Remembering the main features of the canonical solution</th>
<th>Factor 1</th>
<th>Factor 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Conceptual</td>
<td>(Procedural</td>
</tr>
<tr>
<td></td>
<td>knowledge)</td>
<td>skills)</td>
</tr>
<tr>
<td>Item 2: Calculation</td>
<td>-.03</td>
<td>.84</td>
</tr>
<tr>
<td>Item 3: Explaining graphical representation</td>
<td>.64</td>
<td>.03</td>
</tr>
<tr>
<td>Item 4: Calculation</td>
<td>.22</td>
<td>.77</td>
</tr>
<tr>
<td>Item 6: Explaining error</td>
<td>.63</td>
<td>.27</td>
</tr>
<tr>
<td>Item 7: Explaining error</td>
<td>.60</td>
<td>.41</td>
</tr>
<tr>
<td>Item 8: Sorting graphical representation</td>
<td>.66</td>
<td>-.25</td>
</tr>
</tbody>
</table>

To assess differences between the experimental conditions, we calculated a MANCOVA with the
factor condition and the covariate prior knowledge (i.e. previous grade in mathematics) that revealed significant
differences between the conditions for both, procedural skills and conceptual knowledge ($F_{procedural}[3, 139] =
3.45, p = .02; F_{conceptual}[3, 139] = 11.50, p = .00). Mean scores and standard deviations are displayed in Table 3.
We calculated three a priori contrasts: First we compared both Productive Failure conditions to both Direct
Instruction conditions to analyze the effect of the sequence of the problem-solving phase and the instruction
phase. Second, we compared the Productive Failure conditions with each other to assess the effect of the
cognitive prompts in the PF+ condition. Third, we compared the two Direct Instruction conditions, that is, the
different forms of instruction with each other. Concerning procedural skills the a priori contrasts revealed only
effect with a small effect size: The Direct Instruction conditions outperformed the Productive Failure
conditions ($F[1, 139] = 7.02, p = .01, \eta^2 = .05$); differences between PF+ and PF ($F[1, 139] = 3.63, p = .06$) and
between DI-S and DI ($F[1, 139] = 0.14, p = .71$) were not significant. For conceptual knowledge the a priori
contrasts revealed large and medium effects: The Productive Failure conditions outperformed the Direct
Instruction conditions ($F[1, 139] = 26.67, p = .00, \eta^2 = .15$) and DI-S outperformed DI ($F[1, 139] = 19.0, p =
.00, \eta^2 = .12$). However, there was no significant difference between PF+ and PF ($F[1, 139] = 0.07, p = .79$). For
conceptual knowledge an a posteriori Scheffé test indicated significant differences in the pair-wise comparisons
between DI and all other conditions ($p = .00$ for each comparison with DI), but no significant differences
between DI-S, PF, and PF+. The Scheffé test determined two separate homogeneous subsets: One homogenous
subset comprises DI-S, PF, and PF+ ($p = .36$), that is, the means of these three conditions do not differ
significantly. The second homogeneous subset comprises DI ($p = 1$).

Table 3: Posttest results

<table>
<thead>
<tr>
<th></th>
<th>Procedural skills</th>
<th>Conceptual knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Mean</td>
<td>SD</td>
</tr>
<tr>
<td>DI</td>
<td>19</td>
<td>3.68</td>
</tr>
<tr>
<td>DI-S</td>
<td>40</td>
<td>3.56</td>
</tr>
<tr>
<td>PF</td>
<td>39</td>
<td>2.90</td>
</tr>
<tr>
<td>PF+</td>
<td>46</td>
<td>3.45</td>
</tr>
</tbody>
</table>
Discussion

The posttests revealed three major findings: First, we found that the Direct Instruction conditions outperformed the Productive Failure conditions on items testing for procedural skills, yet with a small effect size. This finding is not surprising: Students in the Direct Instruction conditions who received instruction first solved eight practice problems in the following collaborative problem-solving phase. In comparison, students in the Productive Failure conditions solved only one problem during the problem-solving phase prior to instruction. Against this background, the small effect size (as compared to the effect sizes for the other results) seems to indicate that intensive practice may not be the most important learning mechanism.

Secondly, our results show that regarding conceptual knowledge students in the DI condition were outperformed by students of all other conditions, while we did not find any significant differences between DI-S, PF, and PF+. This finding is especially interesting considering that the most common form of mathematics education is Direct Instruction most likely resembling the DI condition. DI-S, PF, and PF+ have in common the form of instruction: In all three conditions the instruction was based on students’ prior knowledge and misconceptions as they are externalized in student-generated representations. Thus, our results suggest that building on students’ prior knowledge and misconceptions during instruction fosters conceptual knowledge. But this instructional approach requires identifying students’ prior knowledge and misconceptions first. Productive Failure seems to be one fruitful approach for triggering students to externalize their prior knowledge and misconceptions as they generate representations in the process of collaboratively solving problems prior to instruction. The subsequent instruction can then build on these student-generated representations. It should be noted, however, that the representations used in the instruction phase of DI-S, PF, and PF+ were not the very representations produced by our participants in the two Productive Failure conditions. Rather, the representations were typical student-generated representations (taken from pilot studies) that matched the representations most often generated in the Productive Failure conditions. It seems that it is not necessary to use students’ own self-generated representations in order to successfully build on their (typical) prior knowledge and misconceptions. Yet, up to date, it has not been systematically investigated whether using typical student-generated representations during instruction is sufficient. So far research that compared learning with own representations to learning with typical representations has focused on the collaborative learning phase prior to instruction: It seems that during the collaborative learning phase students learn more from generating own representations than studying typical representations (Kapur & Bielaczyc, 2011; Roll, 2011). It remains to be investigated whether this difference between learning with own representations and learning with typical student-generated representations also holds true for the instruction phase.

Our third finding was that students’ collaborative problem-solving prior to instruction enhanced conceptual knowledge equally well in both Productive Failure conditions. This finding suggests that the additional cognitive support that was provided in the PF+ condition is not necessary. This finding matches the results of two studies by Weinberger, Ertl, Fischer, and Mandl (2005). In their studies, providing epistemic support that (similar to our cognitive prompts) pointed learners towards specific aspects of the task did not promote learning, while interaction support in form of a role script did. In a previous study (Westermann & Rummel, 2012) we could fruitfully support students’ interaction with a role script in a delayed instruction setting that was similar to the procedure of our Productive Failure conditions presented here. It therefore seems promising to further investigate the differential effects of different dimensions of support for collaborative learning as proposed in the framework by Diziol and Rummel (2010).

Even though the posttest results of the two Productive Failure conditions did not differ significantly, we are interested in finding out whether the problem-solving processes in the two Productive Failure conditions differed, that is, whether the cognitive support had any impact on the learning process. The currently ongoing analysis of our process data will help us to shed light on this question. One aspect is the impact of the cognitive prompts in the PF+ conditions on the number and quality of student-generated representations. A recent study by Wiedmann, Leach, Rummel, and Wiley (2012) indicates that the learning outcome may depend more on the total number of generated representations than on their quality (see also Kapur & Bielaczyc, 2012). In other words, it seems that students should externalize their prior knowledge by generating a large number of representations, including those representations that are based on misconceptions. If this finding was replicated by our data, it could at least partly explain why the cognitive support had no added benefits: The cognitive support probably helped students to improve the quality of their representations, but most likely did not influence the number of generated representations.

Another aspect is to investigate how students collaboratively generate and learn with different representations. Specifically, we focus on the following questions: Do those students who introduce ideas for generating representations during collaborative problem-solving benefit more from this phase than the other group members? Literature points at the importance of active processing of learning relevant representations for learning (Atkinson & Renkl, 2007), but active processing may not require to generate one’s own ideas. Indeed, this assumption is supported by our posttest data: Students in the DI-S condition benefited equally well from learning with typical student-generated representations as students in the Productive Failure conditions who first
generated own representations. To investigate the assumption that collaborative learning can foster elaborative discussion and co-construction (Moschkovich, 1996), we analyze whether students co-construct representations and how this might be related to their learning outcome. The literature on learning with multiple representations (e.g. Ainsworth, 2006) indicates that making connections across representations might be especially fruitful for learning. At the same time, the literature points at students’ difficulties to make connections across representations. We therefore analyze whether and how students make connections across the representations that they have generated themselves.

In summary, the findings from our posttest data support the notion that it is central to build instruction upon students’ prior knowledge and misconceptions. The results of the ongoing process analysis will give further insights into the potential impact of the prompts in the PF+ condition on the collaborative learning process, and shed light on how students collaboratively learn with self-generated representations, that is, representations that exhibit their prior knowledge. At the conference, results of the process analysis will be presented and discussed along with the quantitative results.

References


