Establishing a Mathematical Practice in a Middle School Classroom

Marta Kobiela, Richard Lehrer, Vanderbilt University, Department of Teaching & Learning
0230GPC, 230 Appleton Place, Nashville, TN 37203-5721
Email: marta.a.kobiela@vanderbilt.edu, rich.lehrer@vanderbilt.edu

Abstract: In this paper, we investigate how the practice of mathematical defining was established in one middle school classroom. Our investigation of establishment involves a close look at the co-constitution of practice with communal knowledge. To do so, we employ a theoretical framework that characterizes forms of participation in defining, and illustrates how these aspects contributed to developing a mathematical system of definitions. The teacher, taking the role of a disciplinary representative, supported this co-establishment by modeling aspects of definitional practice, by positioning students as participants in practice, and by positioning definitions at the forefront of discussion. As students began to appropriate definitional practices, they, in turn, became supporters of their collective enterprise of defining. These characterizations of definitional practices and supports may provide resources for developing similar classroom environments.

Introduction

Recently, reform efforts in mathematics education have attempted to provide students with opportunities to participate in mathematics in ways that more closely reflect practices in disciplinary mathematics (Lampert, 1990). Central to these efforts is how such practices are established within classrooms. In this paper, we attend to the establishment of one particular practice, mathematical defining. Our focus on mathematical defining is motivated by the fact that in many classrooms, definitions are often treated in ways that are counter to how they are treated in the discipline of mathematics. Historically, mathematicians have participated in the co-construction of definitions, and defining often emerged from proving (Lakatos, 1976). Some scholars have thus suggested that we instead engage students in defining as a practice, by providing them with opportunities to make sense of and construct definitions themselves, and, in turn, become authors of definition (e.g., de Villiers, 1998; Zandieh & Rasmussen, 2010). Although such studies provide examples of students’ engagement in the practice of defining, very little has been done to show how the practice is established.

In this paper, we investigate how the practice of defining was established in one middle school mathematics classroom. We take the view that a practice is a recurrent activity structure governed by normative expectations about appropriate forms of participation. Practices are tied to the production of knowledge. The practice of defining, in particular, is tied to (a) the production of definitions, (b) the close examination of the properties of the objects being defined, and (c) the network of relations by which new definitions build on established definitions. Thus, our investigation of establishment involved a close look at the co-constitution of the practice of defining with communal knowledge. Accordingly, we were interested in the following two questions: 1) How are knowledge and the practice of defining co-constituted? and 2) How do participants in the community contribute to, or support, this co-constitution? We were particularly interested in the teacher’s role in initially supporting emergent forms of definitional practice and how students, in turn, became participants in the practice. To attend to these questions, we first present a framework for characterizing the practice of defining in classroom communities. We then use this framework to illustrate how the co-constitution of defining and knowledge was established in three excerpts of classroom interaction.

Characterizing Defining as a Practice

To describe the lens we used to examine defining as a practice in classrooms, we begin by describing from a disciplinary perspective what we mean by mathematical definitions and defining. We then outline a framework for characterizing forms of participation in defining in classrooms, what we refer to as aspects of definitional practice. The first author created this framework by reviewing 19 studies in which researchers instigated and/or studied students’ engagement in defining as a practice. These studies varied in content, context and in the age of the students. The aspects of practice were developed through a method of iterative refinement, using the lens of disciplinary perspectives on definitions and defining to determine what constituted an aspect of definitional practice.

Disciplinary Perspectives on Definitions and Defining

A mathematical definition is a description of the properties of a mathematical object (such as a geometric shape) and the relations among those properties (Polya, 1957). Mathematical definitions are distinct from other mathematical entities – questions, conjectures, axioms, lemmas, theorems or corollaries – because they are the negotiated grounds for mathematical work. Unlike axioms, definitions are contested rather than taken for
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forms of activity, we cultivated students' dispositions toward posing questions and making conjectures. We worked from these embodied experiences and conceptions of space, especially bodily motion, and on everyday forms of argument, especially mathematical definitions of geometric objects. Our instructional design capitalized on students' everyday experiences and conceptions of space, especially bodily motion, and on everyday forms of argument, especially propensities to categorize and classify. For example, we anchored students' learning about polygons to paths they walked (Abelson & diSessa, 1980; Lehrer et al., 1989) and related familiar properties of polygons, such as "straight" sides, to experiences of unchanging direction while walking. Working from these embodied forms of activity, we cultivated students' dispositions toward posing questions and making conjectures. We privileged forms of explanation that were oriented toward the general and that appealed to mathematical system.

A Framework for Analyzing Defining in Classrooms: Aspects of Definitional Practice

Definitional arguments and explanations are used to justify (a) inclusion or exclusion of a definition, (b) inclusion or exclusion of an example of a definition, (c) aspects of qualities of the object being defined, or (d) whether conditions in a definition are minimal. For example, Lehrer and colleagues (Lehrer, Jacobson, Kemeny, & Strom, 1999) describe one child’s argument for the inclusion of an example during her class’s construction of a definition for triangle. The child had constructed a triangle with 3 paper strips, one of which was curved. When the class rejected her example as a triangle, she disagreed, appealing to their collectively constructed definition of "3 corners, 3 sides:” “No. It doesn’t matter. Look [gesturing to the board], it has three corners [gesturing to each vertex] and three sides [gesturing to each strip of paper]” (p. 78). This type of argument is emblematic of those within the discipline of mathematics (Lakatos, 1976) because it takes as evidence agreed-upon definitions. Arguments and explanations may take similar forms, but, as illustrated in the example above, unlike explanations, arguments arise from contest and are used to resolve that contest. This distinction is significant because historically the need to resolve disagreements led to advancement in the field (Lakatos, 1976).

Defining also involves the construction and/or evaluation of examples and/or non-examples of the object being defined, where evaluation involves determining whether or not a case should be included as part of the set in question. Constructing and evaluating examples is significant to the practice of defining because it helps students consider what the class of objects being defined should include and provides a set of objects to describe (see, for example, Zandieh & Rasmussen, 2010).

Defining may also involve revising definitions to serve the needs of the mathematical classroom community. Revision often results from definitional arguments or from evaluating examples or non-examples. When revising, definitions are sometimes expanded to include additional properties or relations while, at other times, reduced to become more minimal. In other instances, definitions are instead modified, mainly to improve their correctness.

Proposing definitions about the properties to include in a definition is another aspect of defining. Proposed definitions may then be tested, for instance, against examples of the object being defined, and possibly revised.

Finally, defining involves asking questions about definitions or about the qualities, properties or relations of the objects being defined. Some definitional questions are general (e.g., what is a polygon?). Others are more about particular qualities of an object, often asked in the process of trying to make sense of examples (e.g., “Will this still be a rectangle if I make these sides longer and longer and these shorter and shorter?” from Lehrer, Randle, & Sancilio, 1989, p. 166-167). Questions may also be asked about which properties of an object are necessary and/or sufficient for inclusion in the definition, such as, “Does it really guarantee that if a triangle has two equal angles then it is isosceles?” (Borasi, 1992, p. 34).

Methods

We present data from video records of whole class activity where sixth-grade students created and refined mathematical definitions of geometric objects. Our instructional design capitalized on students’ everyday experiences and conceptions of space, especially bodily motion, and on everyday forms of argument, especially propensities to categorize and classify. For example, we anchored students’ learning about polygons to paths that they walked (Abelson & diSessa, 1980; Lehrer et al., 1989) and related familiar properties of polygons, such as “straight” sides, to experiences of unchanging direction while walking. Working from these embodied forms of activity, we cultivated students’ dispositions toward posing questions and making conjectures. We privileged forms of explanation that were oriented toward the general and that appealed to mathematical system.
Although our focus on spatial mathematics was informed by the school’s grade-level standards for mathematics, the conduct of any particular class was informed by our interpretations of students’ questions and by our judgments of their current levels of understanding.

Participants, Setting and Data Collection
Participants (n=18, 10 male, grade 6, ethnically diverse) attended an urban school serving primarily underrepresented youth in the southeastern region of the United States. Half of the students came from traditional classrooms that emphasized procedural mathematics. The other students had been with the classroom teacher the year before, and had engaged in some conversations about definitions related to mathematical symmetries. Despite this, we still considered the context to be good for studying establishment of practice because (a) norms surrounding participation in practice still needed to be established for new members, (b) old members varied in their participation in practice, and (c) the content was not trivial to students.

Our participants came from a contained classroom, that is, they remained in the classroom with one teacher for all their core academic subjects. The second author served as a visiting classroom instructor for mathematics during the school year, and the regular classroom teacher occasionally interacted as well. The first day of instruction occurred during the second week of school, after a week working within a Connected Mathematics Project (CMP) curriculum unit on polygons. When the visiting mathematics instructor (who we shall from here on refer to as the teacher) first visited, he intended simply to have a conversation with the students about what they had learned. It was only after this first class, when it was clear that students’ ideas about polygons were still developing, that he decided to continue to teach mathematics. Mathematics class was conducted twice each week, for 1.5 hours per class. We videotaped each lesson and then digitally rendered the video for further analysis. We also took field notes of whole group interactions in order to contextualize the video recordings, serve as a platform for reflection, and guide the next day’s instruction. Students also wrote summaries of their thinking at the end of every lesson and took periodic assessments, and both were additional sources of data.

Analysis
For our analysis, we traced initial explorations that emerged as students pursued the question, “What is a polygon?” We focused on the first six days of instruction because the activity largely involved defining and because it allowed us to see how initial forms of definitional practice arose and were supported. To do so, we divided the data into definitional episodes – segments of (possibly overlapping) time in which the class participated in making sense of one particular object (e.g., polygon or side). We limited definitional episodes to whole class discussion in order to capture collective activity. When creating definitional episodes, we identified three 10-minute excerpts of class discussion for careful analysis of the establishment of the practice of defining. We chose the excerpts (from days one, four and six) because they were similar in activity structure (open-ended construction of definitions) and topic (all began with the question, “what is a polygon?”) and served as good representations of shifts in classroom interaction. We wanted the excerpts to be long enough to span multiple definitional episodes, in order to see the development of the mathematical system, but short enough to look carefully at interaction. The excerpts were then transcribed, taking into account both talk and gesture.

We then conducted four phases of analysis. First, we created a representation of the development of collective knowledge as a mathematical system. To create this representation, we looked across neighboring definitional episodes to identify moments of talk, gesture and inscription about interrelationships between mathematical objects and/or qualities of objects. For instance, defining “polygon” created the need to establish what a “side” was, suggesting a link between “polygon” and “side.” Our intention in making this representation was not to make claims about what individuals were thinking, but rather to represent the terrain investigated by the class. Second, using our theoretical framework, we coded when a member of the classroom community (teacher or student) participated in an aspect of definitional practice, using one or more speaker turns as the codable unit. Third, we mapped uses of aspects of definitional practice onto the representation of the mathematical system. Finally, we characterized patterns of interaction within each excerpt in relation to the map between the coded aspects of definitional practice and the mathematical system, and then looked for shifts in these patterns across the excerpts. In particular, we considered the roles taken on by students and the teacher in these interactions. Our choices for determining their roles were guided by the lens of participant frameworks (Goffman, 1981), and in particular O’Connor & Michaels’ (1996) framing of revoicing as positioning. We chose to use this framework because we were interested in how the class’s activity might be positioned as defining and how participants might be positioned as definers. To do so, we looked at how participants (both teachers and students) used talk and gesture to position their collective activity and roles within that activity.

Establishing the Co-Constitution of Practice and Knowledge
Here, drawing upon the three excerpts, we highlight a few ways in which defining and the construction of a mathematical system co-emerged and how that emergence was supported. First, definitional questions served to
encourage the investigation of new and related mathematical objects, and thus supported development of a mathematical system. That is, when a new object was introduced, the teacher asked the students for the definition of the new object. For instance, when the class was making sense of a definition containing “angle,” the teacher asked, “What makes an angle again?” The teacher often further highlighted the importance of new objects by writing the names of the objects on the board. Later, on the fourth day, students began to appropriate these types of questions. For example, after students revised their definition of polygon to include not only “sides” and “angles,” but also “closed,” the teacher asked, “if we take this definition, can there be a polygon with two sides?” One student, Kate, suggested that as long as the two sides were connected, it was possible, and then suggested an oval as an example. When Kate’s example caused many in the class to protest, a group of students asked their peers, “What’s a side, people?” By asking definitional questions, students were beginning to take on the role of supporting one another in their collective activity.

The teacher also played a large role in modeling aspects of definitional practice. As time progressed, the teacher modeled different aspects in order to serve the emergent needs of the community. Initially, as noted above, the teacher modeled the asking of definitional questions that supported development of the mathematical system. Later, he also modeled constructing definitional arguments, and, in doing so, encouraged preciseness in students’ definitions. For instance, when students defined a polygon as having “sides” and “angles,” the teacher drew three connected, but not closed lines, and said, “I want to know what makes something a polygon. I know it has sides and it has angles SO…this then is a polygon right?” In making his argument, he positioned the counter-example in relation to their definition, and, in turn, caused students to revise their definition to include the property of connectedness. In the last class, the teacher’s modeling of definitional practice shifted to address new mathematical relations. For instance, the teacher asked a new type of definitional question, one that encouraged students to think about the economy of their definition: “Can you make any closed figure with sides that does NOT have angles?” At the same time, students continued to appropriate forms of participation that the teacher had been modeling. For instance, in response to the teacher’s question, one student, Ned, constructed the example of a football-shaped figure. When asked to explain his thinking, he pointed to the lines and noted, “two sides,” then pointed to the vertices and said, “no angles.” He continued, “They can’t be angles cause an angle has to be a straight line, two straight lines make an angle.” What is noteworthy about Ned’s definitional argument is that it appealed to his conceived definition of angle in a similar manner as had been earlier modeled by the teacher.

Finally, the teacher also played a large role in positioning both students and content. Initially, the teacher positioned students as participants in aspects of definitional practice. For instance, when one student suggested that a polygon “has the same angles and the same length of uh, same lengths of sides,” the teacher voiced the student’s utterance as a “claim,” thereby positioning his activity as proposing definitions. Another student, in response, suggested, “all regular polygons.” The teacher referred to this suggestion as an “amendment,” in turn positioning her contribution as participating in revising definitions. Later, as the class developed a need to remember their agreed upon definitions, the teacher positioned definitions at the forefront. For instance, when students proposed definitions, he wrote them on the board, and when those definitions were revised, he indicated those changes as well. He also often also requested that students write agreed definitions in their notebooks.

Discussion
In this paper, we provided an illustration of the initial establishment of a mathematical practice, defining. We do not mean to claim that by the end of the six days, the practice was fully established. Rather, we illustrate how in establishing this practice, the roles of the teacher and the students were constantly shifting as the students gained more authority and began to appropriate forms of participation. Our analysis suggests the importance of the teacher in modeling aspects of definitional practice, in initially positioning students as participants in those aspects, and in positioning definition at the forefront of discussion. As students began to appropriate particular forms of participation, the teacher in turn modified what he modeled and positioned to fit the new goals of the community and to support investigation of new mathematical properties and relations. Controversies about definition led to elaboration of mathematically important ideas such as side, angle, polygon, and straight that contributed to the development of a mathematical system. These ideas were then taken up and used during the remainder of the year. Figure 1 illustrates the relation between students’ engagement in aspects of practice, teacher supports and the development of a mathematical system.

Our paper has two contributions. First, the use of our framework of aspects of definitional practice illustrates a potentially significant analytic tool for characterizing student engagement in the practice of defining. This framework has the potential to be refined and expanded as it is used in relation to new classroom environments. Although others have parsed mathematical practices tied to particular content (Cobb, Stephan, McClain, & Gravemeijer, 2001), this paper illustrates how this may be done in regards to an epistemic practice that spans mathematical content. Likewise, the framework, along with the supports we identified, have the potential for supporting teachers interested in developing similar learning environments and supporting students.
in engaging in the practice of defining. The aspects of definitional practice may allow a teacher to identify what types of activity to model and encourage with her students. We focused on collective activity, but this framework may also be useful for capturing changes in how individual students participate in the practice of defining and develop identities as definers. In our ongoing analysis, we are investigating how roles of individual students shift, taking into account their particular histories within the classroom community.

References