Learning Progressions, Learning Trajectories, and Equity

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Abstract: Learning progressions (LPs) and learning trajectories (LTs) are an approach to research that can guide the development of coherent, integrated curriculum, instruction, and assessment. They trace the development of student ideas as they grow in sophistication, across levels that are reasonably coherent networks of ideas and practices. This paper examines whether and how published LPs and LTs address issues of equity. It presents a case study where a curriculum guided by an LP closed or reduced gaps between mainstream and minority groups. We argue that implicit in most LP/LT work is a definition of equity as equal treatment for all students, and propose an alternate definition involving responsive instruction and materials that contemplate individual differences. From the analytic literature review and case study, we abstract guidelines for the development of LPs and LTs, and curriculum materials based on these, to make them more responsive to students and thus more equitable.

Introduction

Learning progressions (LPs) and learning trajectories (LTs) have become influential ways of conducting and synthesizing research to guide the development of science and mathematics curriculum, instruction, and assessment (Duschl, Maeng, & Sezen, 2011). From the outset, three design challenges were identified in the development of LPs: describing students' knowledge and practice at different points in an LP, showing how students can realistically and in connected fashion progress across levels, and "describing the variety of possibilities for meaningful learning for students with different personal and cultural resources or different instructional histories" (National Research Council, 2007, p. 221). Of these three challenges, the issue of equity and diversity was at that time "the challenge we [were] farthest from responding to effectively with the current research base." (NRC, 2007, p. 222). In this paper, we examine the current state of equity in LP/LT research through a comprehensive analytical literature review. We also present a case study of a teaching experiment that closed or reduced gaps between mainstream and minority groups, that used a curriculum based on an LP. From the analytic literature review and case study, we abstract guidelines for the development of LPs and LTs, and curriculum materials based on these, to make them more responsive to students and thus more equitable.

Theoretical Framework

We adopt a cognitive constructivist perspective, in which individuals construct their understanding through interaction with the physical and social environment, by building upon their prior knowledge (e.g., Steffe & Gale, 1995). This perspective owes much to Piaget's work (e.g., Piaget, 1983) although it revalues the role of context and does not adopt Piagetian stage theory. This perspective was explicitly adopted in the original formulation of the LT (Simon, 1995) and has been retained in most published LT papers. Cognitive constructivism is implicit in, and compatible with, the LP literature as well. The first policy report discussing LPs, *Taking Science to School* (NRC, 2007) builds on the seminal NRC report *How People Learn*, which synthesizes research bases including cognitive constructivism, sociocultural theories of learning, expert-novice studies, and cognitive psychology (NRC, 1999). In addition, the cognitive constructivist perspective is compatible with our definition of equity in education outlined below.

Learning Progressions and Learning Trajectories

LPs are "descriptions of the successively more sophisticated ways of thinking about a topic that can follow one another as children learn about and investigate a [science] topic over a broad span of time (e.g., 6 to 8 years)" (NRC, 2007, p. 214). The LP approach addresses the gap between research and practice in education by synthesizing fragmented, small-scale research studies into a form that is meant to be informative to educators and curriculum designers (Duschl et al., 2011). Being organized around core disciplinary ideas and/or practices, LPs provide an answer to the "mile-wide, inch-thick" curriculum (Schmidt, McKnight, & Raizen, 1997) that results from trying to meet standards that are primarily extensive lists of factual knowledge with little prioritization (NRC, 2007). LPs also address the fragmentation of knowledge that results from the use of modules or kits that are meant to be independent and can be sequenced in a variety of ways (Duschl et al., 2011). The first LPs were commissioned papers constructed by groups of science education experts, and dealt with evolution and the nature of matter as well as assessment (respectively, Catley, Lehrer, & Reiser, 2005; Smith, Wiser, Anderson, & Krajcik, 2006; Wilson & Bertenthal, 2005).

LTs originated in the mathematics education community roughly a decade prior to LPs. Simon (1995) defined three components of hypothetical LTs (HLTs): the learning goal, learning activities or tasks, and a model of students' thinking and understanding. Published LTs have tended to use this same definition or a similar one, and the time frame for LTs is usually much shorter than that proposed for LPs – a few classes or a unit (with exceptions, e.g., Confrey, Maloney, Nguyen, Mojica, & Myers, 2009). HLTs are iterative and malleable, the "actual learning trajectories" traversed by students vary by individual, and the teacher is involved in their creation and revision. Recently, LTs and LPs have begun to converge, with LTs seen as finer-grained components of LPs (Stevens, Delgado, & Krajcik, 2010; Plummer & Krajcik, 2010; Battista, 2011).

Equity

Arguments for equity in education include both instrumental and intrinsic rationales (Klasen, 2002). Greater equity in education is an intrinsic goal – it is good in and of itself, on ethical grounds. It is also an instrumental goal: it serves the purpose of maximizing human capital and thus improving the economic competitiveness of a nation (Klasen, 2002). Over the years, there have been many definitions of equity in education (see Kahle, 1996). The first view, prevalent during the Sputnik scare, is purely instrumental, and involves providing the most resources to those students who are most likely to achieve. This view is consistent with "tracking" and other strategies that seek to maximize the learning of the most capable. Other views stress the intrinsic argument, but propose different strategies. A second definition, dating from the civil rights era, views equity as consisting of equal treatment for all students (Kahle, 1996). This view is consistent with the view that "a rising tide raises all ships", e.g., improved curriculum will adequately address equity concerns by improving education for all. A third, postmodern view of equity also stresses the intrinsic argument but acknowledges existing inequities in society. Thus, it proposes responsive, individualized attention to students in order to compensate for past lack of opportunities and to promote social justice.

Applying similar treatment to all students is not likely to result in equality of outcomes for equally capable students, given an uneven playing field. For example, African American and Hispanic students in the USA are three times as likely to be economically disadvantaged (i.e., qualifying for free/reduced lunch programs) and their average achievement on the National Assessment of Educational Progress is around 25 points lower (on a scale of 0-500) than their non-Hispanic White and Asian American peers in math and reading achievement (Hemphill & Vanneman, 2010; Vanneman, Hamilton, Baldwin Anderson, & Rahman, 2009). US females are outnumbered by their male counterparts in STEM fields and express less interest in pursuing STEM careers in college (Hill, Corbett, & St. Rose, 2010). Addressing these issues cannot be done simply by applying the same treatment to all students. Thus, we subscribe to the third definition of equity above. We feel that a definition of equity that involves being responsive to students' individual needs is congruent with a cognitive constructivist learning theory, which proposes building on each individual's prior knowledge.

We organize our study around the following research questions:

1. To what degree do extant LPs and LTs explicitly consider issues of equity?
2. What characteristics of an LP/LT and of the curriculum developed based on the LP/LT might be important in closing achievement gaps between ethnic/racial groups?

Methods

Analytical Literature Review

By means of the analytical literature review we address our first research question. The papers analyzed here propose LPs and LTs for disciplinary content and/or processes. They came primarily from journal special issues and conferences: the journal *Mathematical Thinking and Learning* (vol. 6 number 2, 2004), *Journal of Research in Science Teaching* (volume 6 number 6, 2009), and the 2009 conference *Learning Progressions in Science Conference, Iowa City, IA, June 2009*. We also searched leading journals in science and mathematics education for additional papers on LPs and LTs. LT papers examined included Battista, 2004, 2011; Confrey et al., 2009; Gravemeijer, Bowers, & Stephan, 2003; McGatha, Cobb, & McClain, 2002; Rousham, 2003; Sarama & Clements, 2009; and Steffe, 2004. LP papers included Stevens et al., 2010; Talanquer, 2009; Adadan, Trundle, & Irving, 2010; Smith et al., 2006; Claesgens, Scalise, Wilson, & Stacy, 2009; Merritt, 2010; Lee & Liu, 2010; Alonzo & Steedle, 2009; Duncan, Rogat, & Yarden, 2009; Duncan & Tseng, 2011; Roseman, Caldwell, Gogos, & Kuth, 2006; Songer, Kelcey, & Gotwals, 2009; Catley et al., 2005; Mohan & Anderson, 2009; Mohan, Chen, & Anderson, 2009; Plummer, 2009; and Plummer & Krajcik, 2010.

We qualitatively analyzed each LP/LT paper along three dimensions. Influenced by Rodriguez's (2005) critique of color-blind language in the US *National Science Education Standards* (National Research Council, 1996), we decided to examine the demographics of the participants in each LP/LT paper. We also looked for specific mention of equity issues (e.g., differences in the LP/LT by group), and whether *all* students' ideas were addressed. Additionally, based on a critique by Salinas (2009), we looked for evidence that the LP/LT considered "funds of knowledge" (Moll, Amanti, Neff, & Gonzalez, 1992). The term funds of knowledge refers to the knowledge that students bring to the classroom through their home and community experiences. Students gain this knowledge voluntarily, through their interests and questions. From a cognitive constructivist perspective, considering students' funds of knowledge is essential. Thus, in investigating our first research question, we examine the composition of the students involved in the research for each LP/LT, specific mentions of equity issues and *all* students' ideas, and the use of students' funds of knowledge.

Case Study

Through the case study, we address our second research question. The case study involves a teaching experiment conducted in a free, two-week summer nanoscience camp for 31 middle school students from a diverse, low SES public school district (described in Delgado, 2009a, 2009b; Delgado, Short, & Krajcik, 2009). One of the curriculum strands focused on size and scale, and was developed based on a learning progression (Delgado, 2009a). We analyzed and compared the pre- and post-camp achievement of two groups of students who agreed to participate, were attending the camp for the first time, and attended all sessions: seven mainstream or high-performing minority participants (one Asian-American and six non-Hispanic White students), and 17 historically lower-performing minorities (one Hispanic and 16 African-American students; two did not take the pre-test). While different minority groups have different cultural backgrounds, we placed the students into two groups because of the low number of participants from some racial/ethnic backgrounds.

Two dimensions of knowledge were examined: consistency of knowledge across various aspects or ways of thinking about size (ordering, grouping, relative scale, and absolute size), which is a measure of conceptual understanding; and factual knowledge of the size of key objects such as cells, atoms, molecules, the Earth, humans, etc. (using the same four aspects). The means of the two groups were compared to each other, before and after camp. Our reanalysis for this paper used t-tests for normally distributed variables (pre-camp consistency and pre-camp factual knowledge) and Mann-Whitney non-parametric tests for the non-normally distributed variables (post-camp consistency and factual knowledge). Normality of distribution was tested using the Shapiro-Wilk test. We found that there was a statistically significant difference pre-camp between minority and mainstream groups on consistency (3.0 for mainstream, 1.78 for minority, on a scale of 0-5). Mainstream students also outperformed minority students on factual knowledge pre-camp (10.4 vs. 8.5, on a scale of 0-21), but the difference did not reach statistical significance at the 0.05 level. Thus, there was a gap favoring mainstream students pre-camp. After experiencing the curriculum developed based on an LP, however, the scores for consistency were practically identical for both groups (around 3), and factual knowledge increased for both groups but more for the minority students. Thus, this teaching experiment was effective in increasing achievement and closing gaps. Therefore, it is a good case to analyze in seeking to determine why an LP-guided curriculum was effective.

We analyzed the movement of students from pre- to post-camp for both consistency and factual knowledge in order to detect patterns that might provide insights into what aspects of the curriculum might have been most effective in closing gaps across the racial/ethnic groups. We also used the analysis of the learning activities in terms of the levels of the LP each activity addressed (Delgado, 2009a).

Findings

Research Question 1

We examined the extant literature on LP/LTs for the composition of the student participants. The use of a nationally representative group would enable the LP or LT to reflect the ideas of differing groups of students. In most of the studies the students were not very diverse. Several of the studies used predominantly White, rural or suburban, middle class students (e.g., Adadan et al., 2010; Alonzo & Steedle, 2009; Mohan & Anderson, 2009; Mohan et al., 2009; Plummer & Krajcik, 2010). Many of the LPs or LTs omitted this information altogether. Three LPs (Lee & Liu, 2010; Songer et al., 2009; Stevens et al., 2010) did use student participants that were representative across different groups (e.g., achievement level and race/ethnicity) or groups that were traditionally underrepresented (e.g., low SES ethnic/racial minorities).

We found that the current LP literature does not focus on *all* students' ideas. Instead, the focus is only on those ideas that are shared by most students. Like many LPs, Mohan and Anderson (2009) include only one pathway to the upper anchor, based on the existing curriculum. In a subsequent study (i.e., Mohan, Chen and Anderson, 2009), they propose one alternative pathway, with little attention to how different groups of students may vary in traversing these pathways. While Talanquer (2009) laudably identifies a series of commonly held student ideas, he used student ideas that were held by a majority of students: "We focused our attention on those ideas held by a large proportion of students or those that seem to persist at different learning stages" (pg. 2127), in phase 1; in the next phase, he incorporated "general beliefs" identified in prior literature. Notably, Adadan, Trundle and Irving (2010) made sure to explore all of the alternative ideas of students in their LP development. It could be argued that the large grain-size and temporal scope of LPs precludes detailed descriptions of student ideas, but we argue that ignoring or neglecting ideas that are not in line with the mainstream way of thinking, discriminates against those students. Most LP authors raise few or no issues of equity in their publications. This is justifiable from a definition of equity as equal treatment, but not from our point of view.

LTs stem from an explicitly cognitive constructivist paradigm (Simon, 1995). Furthermore, a model of student thinking is a component of LTs. Nevertheless, the LT literature does not always pay close attention to diverse student ideas. For example, McGatha, Cobb, and McClain (2002) state that they excluded the ideas of some students that differed from the expected and dominant ideas. Steffe (2004) constructed an LT for commensurate fractions for two students, Jason and Laura, who displayed different abilities in partitioning objects. Jason has an *equi-partitioning scheme* that allows him to advance further than Laura, who relies on Jason's explanations to solve tasks. The author maintains that Laura's partitioning and iterating were not part of the same mental structure, as they were for Jason. However, Laura's estimates of a fraction of a stick were on repeated occasions "uncannily accurate" (p. 133). The proposal of one path (Jason's) to an understanding of commensurate fractions, and the way in which Laura's accurate estimates are not seen as suggestive of an alternative, productive pathway, illustrate less responsiveness to student ideas than would seem desirable. (See also Baroody, 2004). In our view, the case of Laura merits closer attention, as a potential alternative pathway to understanding of commensurate fractions. Closer attention to student ideas would be desirable in any LP or LT.

The vast majority of the LPs and LTs did not consider the type of knowledge that students bring to the classroom from their community or family experiences. One exception was Plummer and Krajcik (2010), who specifically talked about including students' "observations of the world, and cultural interactions" when developing the lower anchor of their LP. Songer, Kelcey & Gotwals (2009) mentioned inclusion of student knowledge. In the case of Laura discussed previously (Steffe, 2004), a possible explanation for the uncannily accurate estimates of fractions could be experiences at home. For instance, if her grandmother happened to bake and sell cakes to neighbors, Laura may have had substantial experience slicing cakes accurately into the number of slices requested by a client. Alternatively, if her older sibling had a hobby or business constructing birdhouses, then Laura might have observed or participated in the sawing of long boards into shorter pieces of even lengths. Such concrete wisdom might not be as abstract or generalizable as the coordination of partitioning and iteration schemes, but it might be the kind of prior knowledge that an LT for fractions could be built on, possibly leading to an entirely different pathway to the same goal.

Research Question 2

We represented the observed changes in consistency (i.e., conceptual knowledge) from the teaching experiment from Delgado (2009a) in a new graphic form to better visualize these changes. See Figure 1. Each arrow represents the initial and final state of conceptual knowledge of a single participant. The start of the arrow represents the initial level, and the head of the arrow the final level. Vertical arrows represent students who had no change in their consistency of knowledge, arrows pointing to the right represent students whose consistency increased and arrows pointing to the left represent students whose consistency decreased. We found that the LP-guided curriculum was most effective at helping students under level three, even though most activities were designed to help students achieve levels four and five, involving the coordination of relative scale and absolute size (e.g., realizing that if object A is X times longer than object B, and the absolute size of one object is known, then the absolute size of the other object can be calculated). One instructional activity involve using the optical microscope to successively visualize (at different magnifications): a hair (~0.1 mm in thickness) atop a thin plastic ruler with millimeter markings (1 mm between marks); cheek (~30 μm) and skin cells (~15 μm) next to the hair; and Staphylococcus aureus bacteria (~1 μm) next to the cells. The strategy of visualizing successively smaller objects obeyed general pedagogical strategies, such as going from the concrete and familiar to the abstract and unfamiliar, and to gradually build skills using the microscope. But it concomitantly resulted in an ordered series of objects with successively smaller absolute sizes (supporting Level 3 knowledge, involving the coordination of absolute size and ordering) and larger relative scale factors compared to the reference object of the hair (supporting Level 2 knowledge, involving consistency across ordering and relative scale).

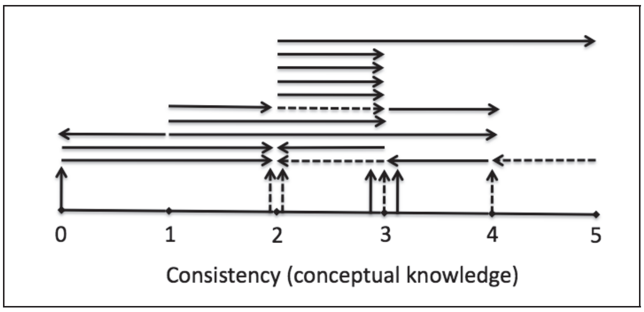

Figure 1. Changes in student consistency (conceptual knowledge), in the teaching experiment. Each arrow represents the initial and final level for one student. The dotted arrows represent non-Hispanic White/Asian American students. Vertical arrows indicate no change.

A subsequent instructional activity used a custom-built computer simulation that uses images of the same objects (hair, cells, bacteria) but extends down to atoms (see Figure 2). Along with a handout to scaffold students' calculation of the absolute size of objects, this activity too targeted Level 4 and 5 understanding (the connection between relative scale and absolute size). There is a sidebar listing all of the objects by size, grouped into those that can be seen at a given magnification, those that are too small, and those that are too large to see completely. This sidebar was primarily meant to allow students to quickly identify objects, as scrolling over their names resulted in highlighting the image of the corresponding object. However, the sidebar itself scaffolds Level 1 understanding of the connection between ordering and grouping. The sidebar in conjunction with the simulation scaffolds understanding of the link between ordering and relative scale (Level 2), and along with the handout to calculate the size of the objects, scaffolds the comprehension of the relationship between ordering and absolute size (Level 3). By including a macroscopic object, the hair, both the optical and virtual microscope activities help Level 0 students who believe that no objects can exist that are too small to see.

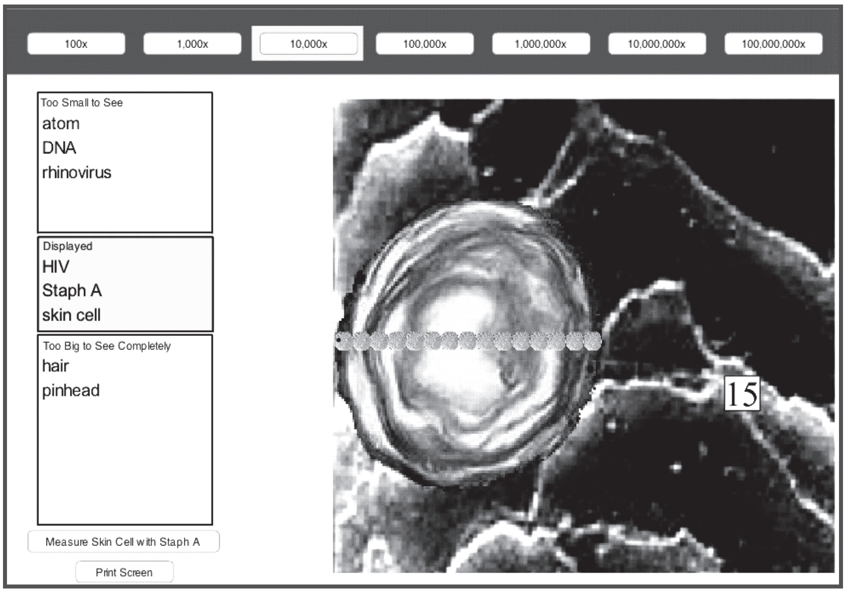

Figure 2. Screen shot of virtual microscope simulation used in the teaching experiment. The sidebar provides ordering and grouping of objects; the main field calculates relative scale and a handout involves absolute size.

An examination of changes in factual knowledge showed that most students, regardless of race/ethnicity or initial level of knowledge, increased from pre- to post-camp. See Figure 3.

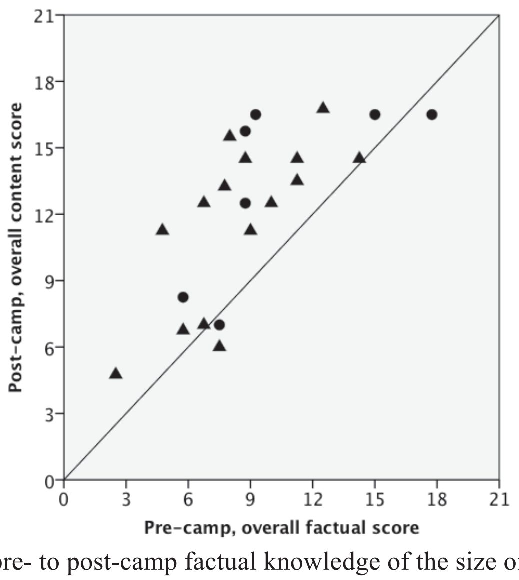

Figure 3. Changes from pre- to post-camp factual knowledge of the size of objects. The diagonal line indicates no change, data points above the line represent an increase. Triangle markers represent Hispanic and African American students, circle markers non-Hispanic White and Asian American students.

Discussion and Recommendations
Based on the analytical literature review and case study, we issue the following recommendations to researchers developing LPs or LTs:

1. Use cross-cultural research to detect a diversity of student ideas. This can be achieved by conducting research in schools with very diverse populations, or in a variety of schools each serving more homogenous groups. The oversampling of traditionally underserved groups would be desirable, in order to ensure that LPs and LTs are responsive to those who face more difficult "border crossings" between home and school cultures (Jegede & Aikenhead, 1999).

2. Make a greater effort to include all students' ideas, rather than just the most common ones. Student ideas closer to the normative scientific or mathematical ideas may be easier to understand, and can be seen more clearly as productive intermediate steps on an LP or LT, but the premise behind LP and LT research is to build on student knowledge.

3. Field analyses may be necessary to get a full understanding of the types of prior knowledge that students bring to the classroom (e.g., Moll et al., 1992). LP and LT developers may have to go into the communities where students reside and get first-hand exposure to the types of informal science and math experiences that students have, so that these can be leveraged in the LP or LT.

4. When designing curriculum units and instructional activities based on an LP or LT, follow a broad-spectrum approach that targets a specific level but simultaneously provides scaffolding for students at lower levels so they may build or reinforce foundational understandings. Project-based and inquiry-oriented instructional approaches already seek to build basic knowledge and skills in the context of student-centered investigations that simultaneously build higher-level cognitive and metacognitive abilities, and may be useful in designing broad-spectrum lessons and units.

Conclusions
These recommendations may seem daunting and time consuming, however, the collaboration between researchers, teachers, specialists, and communities can only serve to produce positive results. If we are serious about promoting equity and serving ALL students, we must be willing to do what it takes to make that happen. Research groups developing LPs and LTs should ideally include advocates for certain groups of students, for example, an expert on special education and team members that are deeply knowledgeable about the culture of minority students. Developing learning progressions and learning trajectories that do not address inequity in educational opportunities in math and science for students will only exacerbate the current problem. As the learning sciences, science education, and mathematics education fields continue to negotiate and define the nature of LPs and LTs, an expansion to include equity concerns at the forefront can greatly benefit groups that have been traditionally underserved.

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