Exploring Connectedness: Applying ENA to Teacher Knowledge

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Abstract: In this study, we consider teacher knowledge of mathematics from the perspective of connectedness. To accomplish this, we adapted Epistemic Network Analysis techniques to characterize the connections between and among pieces of teacher knowledge related to one aspect of proportional reasoning. We discuss the value of this approach as well as directions for further research.

Introduction & Significance

Over the past several years, there has been a growing interest in mathematics teachers' knowledge (e.g., Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008). Despite researchers' efforts to better understand both the nature of teacher knowledge and how that knowledge might shape learning experiences for students, research connecting teacher knowledge to student learning remains inconclusive. For example, Hill and colleagues have found a correlation between teacher knowledge and student growth (Hill, Rowan, & Ball, 2005). At the same time, Baumert and colleagues (Baumert, et al., 2010) were able to statistically distinguish between pedagogical content knowledge and content knowledge in measurements as well as to determine that teachers' pedagogical content knowledge seemed more closely related to student learning than did content knowledge. While these studies suggest progress in understanding specialized knowledge teachers need, other studies present less promising results. For example, Shechtman and colleagues (Shechtman, Roschelle, Hertel, & Knudsen, 2010) did not find differences among student learning accounted for by teacher knowledge of mathematics for teaching (the same construct measured in the Ball et al. studies). Similarly, Kersting and colleagues (Kersting, Giviin, Sotelo, & Stigler, 2002) only found one subconstruct of several to be related to student learning despite their efforts to measure the knowledge that teachers use. In short, more research is needed to understand the nature of teacher knowledge.

To this end, researchers have begun considering how different statistical models, paired with different conceptions of the domain, might help us better understand teacher knowledge (e.g., Izsák, Orrill, Cohen, & Brown, 2010). To further this line of work, in this exploratory study, we use an emerging understanding of teacher knowledge based on research suggesting that the development of expertise is, in part, related to one's organization of knowledge (e.g., Bédard & Chi, 2006; Berliner, 1986; Hiebert & Carpenter, 1992; Leinhardt & Greeno, 1991). This is a departure from studies that consider teacher knowledge from the perspective of measuring amounts of information teachers know. We combine this emerging understanding with a relatively new analysis technique designed to highlight connections among "actors" in a system—whether those actors are people, ideas, or concepts. This approach, Epistemic Network Analysis (ENA; Shaffer et al., 2009), allows the discovery of the interconnected nature of expertise. ENA has been used to understand the processes by which students become more expert-like in game situations.

In the current study, we consider whether ENA can be used to understand the connections among fine-grained ideas about mathematics that teachers use in reasoning about a single proportional reasoning task. We undertake this as a first step toward refining a theory of teacher knowledge grounded in research on expert/novice differences (e.g., Bédard & Chi, 2006; Bransford, Brown, & Cocking, 1999) and related to the knowledge in pieces conceptions of learning (e.g., diSessa, 2006). Our findings highlight the potential benefits of using this approach and we include directions for further investigation.

Theoretical Framework

Expertise & Knowledge in Pieces

Our theoretical framework is grounded in the research on expertise, then introduces Epistemic Frame Theory as the framework of analysis for our study. As mentioned above, we situate this research in prior research on the development of expertise (e.g., Bédard & Chi, 2006) as well as knowledge in pieces (diSessa, 2006; diSessa, et al., 2004). Research on expertise has shown that while experts possess more knowledge of a domain than novices, the quantity of knowledge is not the factor that differentiates them from novices (Bédard & Chi, 2006). Rather, it is the organization of the knowledge that makes them different. For example, experts consider principles rather than surface features in solving problems (Bransford, Brown, & Cocking, 1999). Teachers, for example, differ in their ability to comprehend complex teaching situations in video with the experts more able to attend to important instructional aspects of the class and novices tending to become overwhelmed with all the details of the classroom videos (Sabers, Cushing, & Berliner, 1991).

Developing expertise can be paired with the knowledge in pieces theory (diSessa, 2006; diSessa, et al., 2004), which posits that learners have many fine-grained understandings that are drawn together in different configurations when the learner is presented with a problem. We posit that middle grades mathematics teachers, who are often underprepared in mathematics, may have pieces of knowledge that are present, but not well-connected to other mathematical understandings. For example, in our work, we have seen teachers fail to invoke particular understandings when posed with mathematics problems or student learning situations despite the fact that they have understanding of particular content. As an example, we interviewed one teacher who lamented the inclusion of a “rate” problem about free throws (e.g., which student would you select to make the next free throw based on given data of the students’ recent free throw attempts) in her fractions unit. Despite the teachers’ guide explaining its alignment to the curriculum in terms of equivalent fractions and decimal comparisons, the teacher was unable to reconcile the presence of this task. She knew about fractions and equivalence and she knew about rate, but she could not use those pieces of knowledge together.

Certainly, we recognize that teachers need some amount of particular kinds of knowledge. But, we assert that learning more mathematics is unlikely to be helpful to teachers if they lack *connectedness* in their understanding (e.g., Hiebert & Carpenter, 1992). If teachers are not building connected networks of understanding, they only have a pool of individual ideas, disconnected, and unable to support them (and their students) in the classroom.

Epistemic Frame Theory

Building from the research on expertise, Epistemic Frame Theory (EFT) suggests that complex thinking can be understood in terms of the connections between *frame elements*: different skills, knowledge, values, identities, and epistemological rules from a particular domain. In our case, we are considering particular skills and knowledge related to proportional reasoning.

EFT argues that expertise can be modeled as a network of connections between specific understandings, which are articulated through discourse. Assessing the development of such expertise, however, is a significant challenge. EFT allows us a means, ENA, to quantify the relationships between epistemic frame elements, and thus assess them, using network analysis (Shaffer et al., 2009; Shaffer & Graesser, 2010). Epistemic network analysis (ENA) extends social network analysis by focusing on the patterns of relations among discourse elements rather than individual ideas within that discourse. By treating epistemic frames as cognitive networks, different frame elements—the particular skills, understandings, identities, values and epistemologies of a profession—become *nodes* while the patterns of connections constitute the *links* between these nodes.

By quantifying the patterns of connections between the different elements that combine to form epistemic frames, ENA methods provide a new alternative for measuring complex thinking and problem solving. Thus, we wanted to use them to determine whether they provide insight into the organization of teachers’ mathematical knowledge.

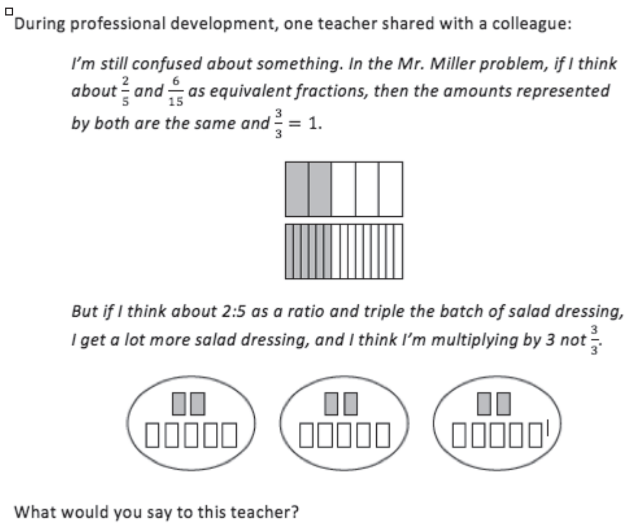

Figure 1: Task presented to interview participants

Methods

The data for this study were collected as part of a larger project aimed at understanding how teachers make sense of proportional reasoning. The data were drawn from a single question (Figure 1) that was included in a clinical interview protocol. The question was from a series of prompts considering the relationship of oil to vinegar in a salad dressing recipe. The videotaped interviews were conducted with middle grades teachers and transcribed verbatim. Those transcripts were coded for this analysis. Note that the researchers had previously coded these data as part of a grounded theory analysis. Therefore, the researchers were familiar with the teachers and their ideas at the outset of this exploratory study allowing coding to be based on the prior coding.

We chose three teachers of varying mathematical ability based on their overall interview (not just the task of interest to this study). The strongest teacher, Lynda (all names are pseudonyms), had been a high school teacher while the other two teachers, Susan and Kate, were middle grades trained. Susan was in the middle in terms of overall performance on the interview tasks and Kate was the weakest. However, all three teachers were clear about their thinking making them ideal for exploring the proposed analysis techniques.

Table 1: Codes used in the data analysis.

<table>
<tr><td rowspan="2">Ratio Concepts:
R1: Iteration
R2: Ideas of Amount/Quantity/ Growth
R3: Invariance
R4: Iteration without Calculation
R5: Ratio-as-Measure
R6: Ratio Calculation</td><td>Fraction Concepts:
F1: Fractions (generic)
F2: Rules for Operations
F3: Equivalence
F4: Comparison</td><td>Problem Solving:
P1: Problem Interpretation
P2: Verify Mathematics is Correct</td></tr>
<tr><td>Representation (Diagram):
D1: Interpretation of Diagram
D2: Validate Representations</td><td>Other Mathematical Ideas:
M1: Equality
M2: Discussion of Fraction/Ratio Relationship
M3: Attending to Context</td></tr>
</table>

Epistemic Network Analysis

We coded the interviews for important ideas related to connections between fraction and ratio ideas as they arose (Table 1). These codes captured ideas about fractions, ratios, problem solving, reasoning with drawings, and other mathematical topics. For this exploratory analysis, we treated each box in Table 1 as a node and used ENA to help us find the linkages between those nodes.

Formally, an epistemic frame can be characterized by individual frame elements, f_i, where i = a particular coded element of an epistemic frame. For any participant, *p*, in any given interview, *s*, each segment of discourse, $D^{p,s}$, provides evidence of whether participant *p* was using one or more of the epistemic frame elements (nodes).

Each segment of coded data was represented as a vector of ones or zeroes representing the presence or absence, respectively, of each of the codes. *Links*, or relations, between epistemic frame elements were defined as co-occurrences of qualitative codes within the same segment. To calculate these links, each coded vector was then converted into an adjacency matrix, $A^{p,s}$, for participant *p* in a given story assignment, *s* (1).

$$A^{p,s}{}_{i,j} = 1 \text{ if } f_i \text{ and } f_j \text{ are both in } D^{p,s} \qquad (1)$$

Each coded segment's adjacency matrix, $A^{p,s}{}_{i,j}$, was then converted into an adjacency vector and summed into a single cumulative adjacency vector for each participant *p*, and each story assignment, $U^{p,s}$, i.e., for each unit of analysis (2).

$$U^{p,s} = \sum A^{p,s} \qquad (2)$$

For each participant, *p*, and each story assignment, *s*, the cumulative adjacency vector, $U^{p,s}$, was used to define the location of the segments in a high dimensional vector space defined by the intersections of each the codes. Cumulative adjacency vectors were then normalized to a unit hypersphere to control for the variation in vector length, representing frequencies of co-occurring code pairs, by dividing each value by the square root of the sum of squares of the vector (3).

$$nU^{p,s} = U^{p,s} \;/\; \sqrt{\sum}(U^{p,s})^2 \qquad (3)$$

This was necessary to emphasize similarities in the types of co-occurring code pairs rather than similarities based primarily on the frequencies of co-occurring code pairs.

A singular value decomposition (SVD) was then performed to explore the structure of the qualitative code co-occurrences in the dataset. SVD was used to first project the normalized cumulative adjacency vectors into a high dimensional space such that similar patterns of co-occurrences between epistemic frame element codes would be positioned proximately. The SVD analysis then decomposed the structure of the data in this high dimensional space into a set of uncorrelated components, fewer in number than the number of dimensions that still account for as much of the variance in the data as possible. The resulting networks were then visualized by locating the original frame elements in a two dimensional space as follows:

For each frame element e_i we compute the mean coordinates $\bar{x}_i$and $\bar{y}_i$ of all of the n^2 loading vectors $V^{i,j}$ for j=1 to n:

$$\bar{x}_i = \frac{\sum_{j=1}^{n} V_x^{i,j}}{n}$$
$$\bar{y}_i = \frac{\sum_{j=1}^{n} V_y^{i,j}}{n}$$

This produces a distribution of nodes in the network graph determined by the loading vectors that contain them in the space of adjacency vectors. Nodes are then positioned in the space, and links are constructed according to the adjacency matrix.

Findings

We found substantial differences between the teachers' network graphs. An initial analysis determined that fraction concepts largely fell to the left of the *y*-axis while ratio concepts fell more to the right. For example, in Kate (Figure 4) we see more attention to thinking about rules for operating on fractions (f2) and finding equivalence (f3). In contrast, Lynda's map (Figure 2) shows more attention to ratio concepts, like ratio as measure (r5; meaning the measure, "taste", is the fixed relationship between the vinegar and the oil) and the idea of invariance (r4; meaning the relationship between the two values remains constant). The *x*-axis appears to divide ideas related to procedural and conceptual understandings. For example, Susan's graph (Figure 3) shows the relationship of fractions and ratios (m2) as well as problem interpretation (p1) below the *x*-axis.

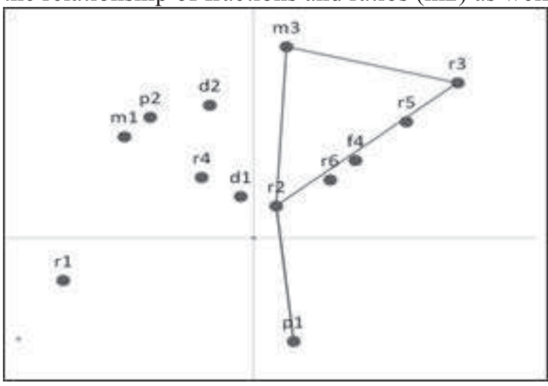

Figure 2: Lynda's map.

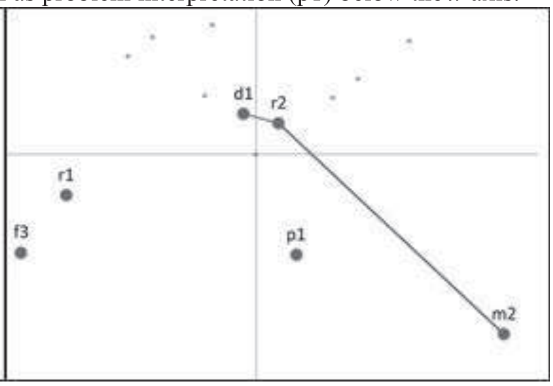

Figure 3: Susan's map.

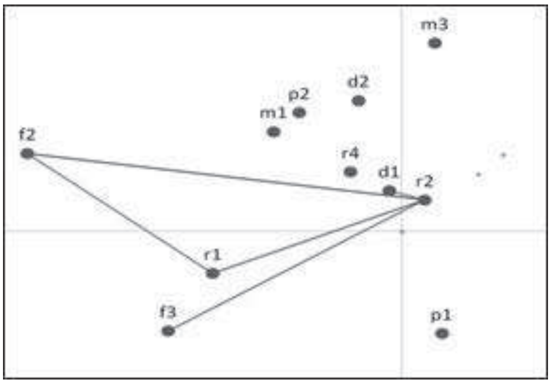

Figure 4: Kate's map.

The maps also show the teachers' connectedness of concepts in response to our prompt. To appear as a point, a concept has to have been connected to another concept at least one time. Concepts with lines connecting are related by at least two connections. For example, in Lynda's graph, ideas of quantity (r2) and attending to the context (m3) co-occurred at least twice, indicating some kind of relationship between those ideas for Lynda.

Analysis of these maps suggests to us that Lynda relies on ratio reasoning and problem solving skills (e.g., understanding the problem) to make sense of ratios. In contrast, Kate relies on fraction rules and operations to make sense of these concepts. Susan, interestingly, leaned more toward ratio understandings, but introduced fewer key ideas together as evidenced by the limited number of points in her graph. We posit that this may be a natural step in the progression toward expertise. Kate, our least expert teacher, introduced many ideas that were not connected, while Lynda, the most expert-like teacher, introduced many ideas that were more closely connected. Lynda may have a high number of conceptual resources from which to draw and more connections among those ideas than Kate. Susan did not invoke as many ideas, but the ideas she did invoke were largely connected, suggesting that connectedness of ideas and number of ideas are not necessarily related in active problem-solving activities, supporting the findings of expertise research that suggest expertise is more than an accumulation of knowledge.

Next Steps

This exploratory study first confirms that fine-grained analysis of teachers' cognition can lead to interesting and informative network maps. Next steps will include the analysis of larger datasets as well as refinements to the coding scheme to disentangle conceptual and procedural understandings. In short, we see ENA as having promise for making sense of teacher understanding, but more work is needed.

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