The Obj–Subj Dialectic and the Co-Construction of Hierarchical Positional Identities During a Collaborative Generalization Task

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Abstract: This paper presents a theoretical model of mathematics teaching and learning that captures the reflexive relationship between processes of objectification and subjectification inherent in classroom discourse. I call this model the "obj–subj dialectic," and ultimately I posit that students' respective semiotic means of objectification also function, simultaneously, as semiotic means of subjectification. I use this model to analyze and describe the individual student learning of three high school students engaged in a generalization task and the emergent social power dynamics. Findings show that whereas students made statements that positioned themselves as having the correct answer (i.e., as mathematical authorities), some students resorted to a broader arsenal of semiotic resources to make their point, which resulted in differentiated status positions. Thus some students gained "mathematical ascendency" over others, which refers to the co-construction of hierarchical positional identities emerging from the obj–subj dialectic during multimodal interactions.

Keywords: discourse, objectification, mathematical cognition, instruction, power dynamic

Introduction: The story of Thalia, Ailani, Xeni, and Mr. Lam

Three high school students, Thalia, Ailani, and Xeni, have been working together on a pattern-finding problem, when their classroom teacher, Mr. Lam, approaches their table and asks questions related to their work. The problem presents a sequence of four geometric figures, and there is a numerical pattern that enables students to make predictions about figures further along the sequence, such as Fig. 10. Taking lead of the group, Thalia has made a prediction about Fig. 10, using an arithmetic strategy that effectively determines figures in the sequence. However, her strategy achieves this goal using a recursive technique, that is, by iteratively progressing from each figure to the next, whereas optimally a strategy would determine figures by algebraic function, without recourse to known items along the sequence. Mr. Lam prompts the group to consider much larger numbers, such as Fig. 100, asking, "So we have to know the one before it [Fig. 100]? Is there a way that we don't have to know the one before it?"

Mr. Lam may not be aware of it, but a conflict is about to erupt. Implicit in his question is the assumption that the group's strategy is inefficient. After a 2-sec pause, Thalia stomps her feet and exclaims to Mr. Lam, "Bruh, why you asking all these questions?" Ailani stirs the conflict, interjecting loudly, "YES!" Mr. Lam, keeping his usual calm and professional demeanor, responds to Thalia: "My name is not 'Bruh.' My name is Mr. Lam, and I'm challenging you. That's why I'm asking you these questions, because I want you to get smarter." Thalia buries her face behind her hands and starts laughing sheepishly; her face still hidden behind her hands, she continues: "OK, Mr. Lam, why you asking (all these questions)."

Background and objectives

There is more to learning mathematics in a classroom than learning content. I recorded the above vignette while observing Mr. Lam's classroom as part of my research on the challenges and opportunities for improving mathematics education for students from historically marginalized public school populations. Thalia's statement of "Why you asking all these questions?" reflects a complex sentiment. She may perceive that she and her peers are being questioned unnecessarily, even interrogated about their participation in the task. Ailani agrees with Thalia's sentiment and, together, their behaviors indicate their sense of Mr. Lam's instruction as coercive at worst, or troublesome at best.

This vignette sparked a variety of questions for me. What does it mean for mathematics students to experience classroom discourse in this way? What aspects of classroom interaction give rise to this type of social-mathematical antagonism? What is the pedagogical utility of framing mathematics learning as adversarial or as changing intellectual capacity?

Thalia and Ailani's apparent discontent with their mathematics teacher and Mr. Lam's particular views on mathematics pedagogy could be isolated phenomena limited to the envelope of this particular classroom community; it is conceivable that this interaction between three high school students, a teacher, and mathematics content is not representative of broader trends. However, by reflecting critically on the results of previous studies that I conducted in similar classroom settings (e.g., Gutiérrez, 2010, 2013), I argue that these participants' experiences are related to broader issues endemic to not only mathematics classrooms serving historically marginalized students, but to all classrooms. The study presented in this paper is part of a larger project that aims to show that mathematical learning is not power neutral, but rather individual learning and power relations are mutually constituted through discourse.

The broader research project investigates how relations of power, which are inherent to all educational settings, impact students' quality of engagement and therefore learning. Whereas I focus my research efforts specifically on students from historically marginalized public school populations, such as African American, Latino/a, and economically disadvantaged students, I theorize that *all* students' participation, dispositions toward classroom practice, and mathematical knowledge are mediated by relations of power that shape local instructional contexts and the social interactions therein. To illuminate these issues, I bring to bear complementary learning-sciences and sociopolitical perspectives to expose hidden structures, mechanisms, and processes of power that either enable or hinder classroom mathematics learning.

The objectives of this paper are (1) to provide a sketch of the theoretical stance underlying the larger project, and (2) to present an emerging approach that stems from this theoretical orientation. At the center of this approach is the notion of an *objectification–subjectification dialectic*, which I define, apply, and elaborate in the sections below. Specifically, I analyze Thalia, Ailani, and Xeni's multimodal interactions during a group task, through the lens of the obj–subj dialectic.

Theoretical perspectives and relevant literature

The larger research project assumes the following theoretical stance: individual learning and social power relations become imbricated through discourse so as to mutually constitute and express each other. I view discourse not merely as an analytic lens for observing the *learning–power imbrication*, but also as the medium through which mathematical knowledge and power relations simultaneously become objectified, stabilized, and reproduced. In particular, I conceptualize the learning–power imbrication as the reciprocal discursive process whereby (1) students appropriate cultural artifacts (e.g., algebraic symbols and forms) as semiotic means of objectifying personal pre-symbolic, proto-mathematical knowledge (Abrahamson, 2009; Radford, 2003); yet so doing, (2) students not only adopt a new perspective on the world but they also become cognitively "beholden" and subject to particular discursive practices that temporally and ontologically precede them (cf. "ontological imperialism," Bamberger & diSessa, 2003) and are inherently hierarchical. Thus, acts of (1) objectification (that forge individual learning) are at the same time acts of (2) subjectification (that reify relations of power). See *Figure 1*, below.

The obj–subj dialectic in mathematical discourse is not a novel perspective per se and similar ones are found in the literature (e.g., Heyd-Mezuyanim & Sfard, 2012; see below). However, returning to the underlying theoretical stance of this line of work, I have found the construct of a learning–*power* imbrication especially useful, because K-12 mathematics education occurs within a broader nexus of asymmetric power relations that have not been adequately accounted for in the literature. The goal of this paper is to explore how the analytic construct of the obj–subj dialectic can shed light on hidden discursive mechanisms that give rise to the learning–power imbrication in mathematics education.

In the model, the "obj" side of the dialectic is based on Luis Radford's theory of knowledge objectification (2003), and in particular his semiotic–cultural taxonomy of students' types of generalizations—*factual*, *contextual*, and *symbolic* generalizations ("F-C-S"). The F-C-S framework distinguishes among three generalization types in accordance with their level of generality (see Radford, 2003, for more detail). I have adapted the framework and applied it in a diversity of educational settings. I aruge that F-C-S represents three *modes of action* (Gutiérrez, 2010) that students appropriate as means of dealing with pattern-generalization problems. In other words, the F-C-S framework describes both the final *products* of students' algebraic reasoning, as well as the *processes* that would result in those products. Thus the F-C-S framework enables analysis of the semiotic spaces that students *and* teachers must navigate. Furthermore, I argue that these semiotic spaces both reflect and influence students' mean-making while at the same time, these semiotic spaces are differentially imbued with status and authority. That is, as students and teachers navigate or traverse different levels of generality inherent in mathematical discourse, they simultaneously navigate/traverse relations of power (subjectification), which I explain next.

Turning to the "-subj" side, I combine several perspectives from various bodies of literature. Most relevant to this paper however, is the work of Anna Sfard. Sfard and colleagues (2012) open up new analytic avenues dealing with *direct* subjectification—when certain verbal utterances directly indicate a human actor. Building on yet extending this approach, I show in my data analysis instances where the referents of verbal utterances do not directly involve human actors but nevertheless involve *indirect* subjectification. Some utterances/actions mark subjectification through tacit positioning (Harré, 2008).

Furthermore, as students make mathematical assertions during collaborative problem solving, I argue, their discursive action creates hierarchical *positional identities* that must then, in turn, either be taken up, accepted, contested, negotiated, or rejected. The construct of "positional identity" is based on the work of Marcy Wood (2013) on "micro-identity" that describes "identities enacted in a moment in time" (p. 778). However, I emphasize the "positional" aspect to describe an emerging power dynamic, and the term already points to the mechanism by which hierarchizing occurs—positioning (Harré, 2008). That is, I conceptualize positional identities as hierarchical, because co-constructed subject positions are differentially imbued with social status and authority. Furthermore, these hierarchical positional identities are associated with students' "locations" along the F-C-S trajectory.

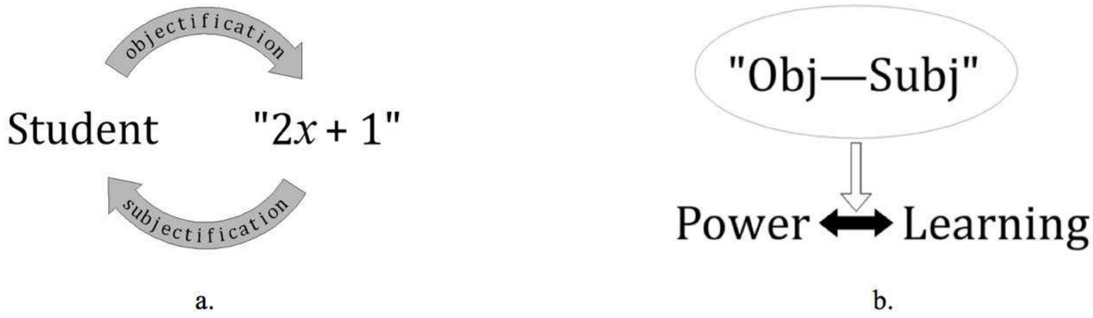

Figure 1. (a) The obj–subj dialectic: a synthesized analytic construct for observing how students appropriate cultural artifacts (e.g., algebraic symbols and forms) as semiotic means of objectifying pre-symbolic knowledge, in relation to subjectification acts that shape their positional identity. (b) As students enter each of the "F-C-S" discourse stratums and shift across them, they simultaneously create positional identities that are differentially imbued with status and authority. Thus, mathematics learning and ways of knowing become imbricated with emergent relations of power.

Methodological approach

Below I analyze two transcription segments from a much longer vignette of video data. Specifically, I explore the tension between "obj," the semiotic resources (e.g., gesture and language, as well as conventional tools such as tables and graphs; Radford, 2003) to which students have recourse to make mathematical assertions, and "-subj," the hierarchical positional identities that participants co-construct through these multimodal interactions (cf. Harré, 2008; Sfard & Prusak, 2005).

Data, participants, and task

The data are from a year-long participant ethnography where I immersed myself within a single mathematics class at César Chávez School for Restorative Justice (all names are pseudonyms), a small public high school located in a large, diverse urban district in northern California. At the time of the data collection (2013-2014) over 90% of the student body was Latino/a, African American, or recent immigrant students—all from the working-class and low-income neighborhoods surrounding the school.

This data collection project combined classroom observation with principles from design-based research. The focal data was collected in the Fall semester during a phase in the project where I worked closely with the teacher, Mr. Lam, to co-design and implement an experimental instructional unit. I focus my analysis on a 35-min span of video involving a group of three female students, Thalia (Grade 9), Ailani (Grade 9), and Xeni (Grade 10). The group assignment is a pattern task that combined geometric objects called "Spiralaterals" with algebraic reasoning. A Spiralateral is drawn on graph paper and is derived from a set of rules, see *Figure 2*. Frank Odds (1973) describes how to construct a {1, 2, 3} Spiralateral:

> First draw a segment of unit length to coincide with the edge of a graph square. Turn right through 90° and draw a segment two units long. Again turn right through 90° and draw a segment three units long. A basic pattern of 1-2-3 has now been established. Repeat the same steps again, continuing from the outer end of the three-unit segment. After [three] repetitions of the basic pattern, the segment will join the point at which the diagram started (p. 121).

The specific task involved a poster that presents the first four figures (i.e., "Fig. 1," "Fig. 2," "Fig. 3," and "Fig. 4") of a Spiralateral sequence, and the task objective was to express a "Code" as a set of algebraic formulas in the form of $\{f_1(n), f_2(n), f_3(n)\}$ and whose inputs are the figures' ordinal positions (*Figure 3*). A key design feature for implementing the Spiralateral sequences was to substitute larger numbers (e.g., Fig. 10, Fig. 100), as a way for students to realize that an arithmetic-recursive strategy may be inefficient, thus motivating the need for more powerful tools and strategies such as algebraic generalizing and the use of direct formulas.

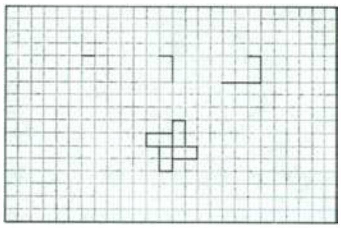

Figure 2. Steps in the construction of a "3-legged" Spiralateral (image from Odds, 1973).

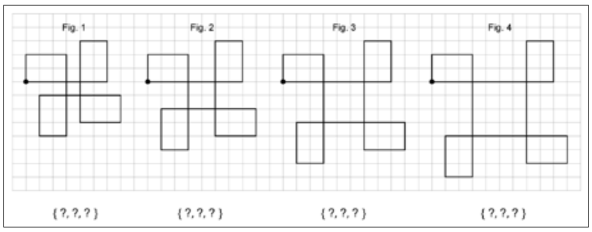

Figure 3. Poster presenting a 3-legged Spiralateral sequence, modeled as {2, 3, n+5}.

Sample data analysis: The obj–subj dialectic as an analytic frame

Here I present two transcription episodes followed by a line-by-line qualitative microgenetic (Schoenfeld et al., 1991) analysis of the semiotic resources in the students' obj–subj processes. In this first episode Ailani adamantly claims that the solution to Fig. 10 is nineteen, whereas Thalia argues that it is fifteen; Xeni is reading her book and does not contribute mathematically in this particular transcript segment. (Note on transcript conventions: double slashes "//" mark beginning and end of overlapping utterances; two dots ".." at end of text is very slight pause, less than a second; repeated letters, e.g., "generaliiize," mark lengthened syllable, each repeated letter equals one "beat"; and "(??)" and "(this)" are unclear/inaudible reading or a tentative reading.)

Episode 4 (of 13) – Timestamp [08:13–09:34] – Thalia, Ailani, and Xeni work on team task

122 Ailani:	[holds poster in front of her face with both hands, counts aloud; as she counts, her right hand comes free and she begins counting with her fingers] "Eleven twelve.. thirteen.. fourteen, fifteen, sixteen, seventeen, eighteen, nineteen." [reaches back and taps Mr. Lam on the shoulder] "Fig. 10 will be (1-sec pause) nineteen." (10-sec pause) [to no one in particular, she announces] "(Fig. 10 will be).. nineteen."
123 Thalia:	[gazes down; indicates table of values] "Fig. 10 will be fifteen."
124 Ailani:	[responds to Thalia] "Nineteen."
125 Thalia:	[gazes down; shakes head no] "Fig. 10 will be fifteen."
126 Xeni:	[leans her head on her hand; unclear where her gaze is].
127 Ailani:	[loudly, almost shouting] "Fig. 10 will be nineteen!"
128 Thalia:	[still without lifting her gaze, indicates entries with her finger as she talks in a rhythmic cadence] "It goes two-three-six, then it goes two-three-seven, then it goes two-three-eight, then it goes two-three-nine, then it goes two-three-ten, then it goes two-three-eleven, twelve, thirteen, fourteen, fifteen." [keeps gaze down, continues working]
129 Xeni:	[adjusts in her seat; it is possible that she responds to something Thalia or Ailani said, but one cannot tell from the footage]
130 Ailani:	[briefly gazes in Xeni's direction, but it is unclear if she actually addresses Xeni; likely addressing no one in particular] "I say nineteen. (4-sec pause) I'm not even sure I counted that right, but who cares?"

Analysis of Lines 122–130 reveals two important components. First, this excerpt reflects Thalia's and Ailani's first mathematical objectifications—both of which were articulated in the Factual mode. Ailani's semiotic means of objectification consisted of a counting strategy and verbal speech (with a certain illocutionary force) to assertively express a partial solution, in the form of $f_3(10) = 19$. Ailani counted on from a known quantity ($f_3(4) = 6$) but she had not yet indicated that she was attempting to generalize a numerical pattern. Thalia too resorted to verbal speech, yet she also used rhythm, gesture, and repetition, and a mathematical table as semiotic resources. Entering a rhythmic cadence (Line 128) enabled Thalia to objectify an arithmetic-recursive generalization, in the form of $\{2, 3, f_3(n-1)+1\}$ which, in this context, is more informative than Ailani's solution.

The second component this transcription reflects is a dynamic process of subjectification. To examine the dynamics of subjectification, I focus on whether/how Thalia and Ailani refer to themselves and each other. Thalia did not refer to herself or Ailani directly. Looking at Thalia's pronominal usage, what we see is that the referents of her speech did not directly involve human actors. And yet, Thalia made statements that implied *she* has the correct answer and *Ailani* does not. Ailani, too, made statements that tacitly positioned herself as having the correct answer. Thalia, however, resorted to a broader arsenal of semiotic resources to make her point. Additionally, Thalia did not make eye contact with Ailani, instead keeping her gaze on her work. This social-mathematical power encounter resulted in differentiated status positions. Ailani's effort to appropriate mathematical authority was challenged, and an opportunity was missed to engage in dialog and collaborate on the shared goal, to determine Fig. 10.

In sum, both Thalia and Ailani attempted to gain *mathematical ascendency* over the other, which is a construct I claim that describes the hierarchical status positions that were being co-constructed in the moment. Mathematical ascendency is based on the semiotic resources marshaled by and pitted against individual interlocutors. Thus, Thalia's and Ailani's respective semiotic means of objectification simultaneously functioned as semiotic means of subjectification.

Importantly, if we had been looking at subjectifying utterances only as defined by Sfard and colleagues, we would highlight only one single incident, when Ailani admits her counting strategy's inaccuracy (Line 130). But analysis of this single turn of talk, although a crucial one, does not capture the dynamics of implicit positioning in Lines 122–129. As the three students continue to make mathematical assertions throughout the remainder of the episode, they co-construct positional identities that must then, in turn, either be taken up, negotiated, or rejected.

<u>Episode 5 (of 13) – Timestamp [09:34–10:56] – Mr. Lam's first visit with Thalia, Ailani, and Xeni</u>

This episode begins as Mr. Lam returns to Team 1 in response to Ailani's tap on his shoulder. Ailani and Thalia once again assert their solutions regarding Fig. 10 and attempt to establish their mathematical ascendency. As they argue, Mr. Lam turns to Xeni and asks her questions that would steer her toward acting in relation to the assigned task. What I aim to show in the detailed analysis, below, is that although Ailani and Thalia argue their respective solutions and appear to be reaching an impasse, their individual learning trajectories nevertheless show movement from the Factual to the Contextual modes of reasoning and, moreover, *that these advancements were forged in and through the social mathematical power dynamic*.

131 Mr. Lam:	[to Ailani] "Alright what's the question? What was the question? You came up with how many—the code?"
132 Ailani:	"Nineteen."
133 Mr. Lam:	"Nineteen?"
134 Thalia:	[loudly] "No! No. [softly, counts to herself; keeps her gaze down] Twelve.. fourteen.. [to Mr. Lam] It'll be fifteen."
135 Mr. Lam:	[walks around to Xeni's side of the table; slides poster closer to her] "Fifteen? //Xeni, what do you think?"
136 Thalia:	//"It'll be fifteen.// I think I did this right but I might be wrong."
137 Xeni:	"(??)"
138 Mr. Lam:	"Hmm."
139 Xeni:	[turns her gaze away from Mr. Lam, down to her desk] "I don't know."
140 Mr. Lam	[to Xeni] "OK, what might help? You think maybe writing in the code might help? [slides poster closer to her and indicates the "{?, ?, ?}" along the bottom of each figure] If you write in the code and look for a pattern?"
141 Thalia:	"It'll be fifteen. [waves a sheet of paper in front of Mr. Lam but he ignores it and keeps his gaze on Xeni; Thalia gazes up at Mr. Lam for the first time, hands a sheet to Mr. Lam] I already wrote all the codes."
142 Ailani:	[gaze down at her desk; she is drawing]

143 Mr. Lam:	[grabs sheet from Thalia and places it in front of Xeni on her desk] "Oh good she's got the codes. So maybe you can find a pattern based off of her codes. [talks in a singsong voice as he points with his fingers to values in Thalia's table] Two three six, two three seven—"
144 Ailani:	[interjects loudly; to Mr. Lam] "It just added a number!"
145 Thalia:	[interjects loudly as well; to Mr. Lam] "It just goes six seven eight nine ten eleven twelve thirteen fourteen fifteen."
146 Mr. Lam:	[nods head, either agreeing or counting along or both] "Two three eight, two three nine, two three ten.."
147 Thalia:	[with a tone suggesting that the pattern should be obvious to Mr. Lam] "Eleven twelve thirteen fourteen fifteen."
148 Mr. Lam:	"So what do you notice about the first two [referring to the first two entries of the code]?"
149 Thalia:	"What do you mean?"
150 Mr. Lam:	"What do you ah—"
151 Thalia:	[makes repeating arching gesture across the figures in the poster] "It goes six seven eight nine ten eleven twelve thirteen fourteen fifteen."
152 Ailani:	[adjusts in her seat; without lifting her gaze, loudly] "No, it don't!"
153 Xeni:	[no indication she is contributing to the converstaion; yet she could be observing/listening]
154 Thalia:	"It does. It always adds//"
155 Ailani:	//[to Thalia] "Bruh, it's like ten [lifts gaze from her desk to Mr. Lam] but you added one or two."
156 Mr. Lam:	[to Thalia, indicating entries in her table of values] "Two three—no but—no but look, it says two then three then //six."
157 Thalia:	//"Six."
158 Mr. Lam:	"Then this one says two then three then seven. // Two three eight.
159 Thalia:	//[with a tone suggesting that the pattern should be obvious to Mr. Lam; talks fast, indicating entries in the table with her finger] "It goes two three and then eight, and then it goes two three and then nine, and then it's going to be two three and then ten, and then eleven twelve thirteen fourteen fifteen."
160 Mr. Lam:	[to Thalia] "OK so two three fifteen, you're saying?" [to Ailani; makes repeated arching gesture with his right hand, marking each "entry" of the code with each motion] "OK that's what she meant when she said fifteen. So that code, it's staying two [gestures up-down], three [gestures up-down], and then something [gestures up-arching over to the right-down, palm up; starts to walk away]. [to the group] OK so how would you find for any—for any number?"
161 Thalia:	[throws hands in the air; posture and facial expression suggest frustration; to Xeni] "You just keep adding the number, duhhh!" [looks to Ailani, then looks to Xeni]
162 A. & X.:	[no response]

Earlier in Episode 4, we observed that Thalia's and Ailani's semiotic means of objectification also served as semiotic means of subjectification, and as such these semiotic resources formed the basis of a social-mathematical hierarchy. Here in Episode 5 the power dynamic continues and the students marshaled their semiotic resources to resolve an overt mathematical conflict, but we also observe that the teacher marshaled his semiotic resources, as well, to orient Xeni toward acting in ways that are relevant to the problem-solving domain. When Mr. Lam walked away from Ailani and Thalia's side of the table to Xeni's side and asked her, "What do you think?" he positioned Xeni as the center of attention and her (non)participation as the critical aspect of the situation, even as Ailani was vying for his attention regarding Fig. 10.

Thalia too fought for Mr. Lam's attention, for him to recognize that she had done all the work, when she stated "I already wrote all the codes" as she waved the sheet of paper in Mr. Lam's view. Her tone of voice suggested that he need not bother Xeni with that task because it was "already" completed. Mr. Lam maintained his usual warm, professional demeanor at this moment as well, and finally accepted Thalia's gesture to look at her codes. In that moment, Ailani interjected with, "It just added a number!" (Line 144), which is an utterance iterated in the Contextual mode as it referred to a general procedure and was not tied to concrete elements in the problem space. What is most interesting to note at this point, is that this is the first student utterance coded in the Contextual mode, whereas all previous utterances were articulated at the Factual level of generality. Thalia, too, interjected at the exact moment that Ailani articulated her Contextual statement (Line 145), with a re-articulation of her Factual recursive generalization: "It just goes six seven eight nine ten eleven twelve thirteen fourteen fifteen." Mr. Lam responded to Thalia's and not Ailani's contribution, asking a question about the pattern that Thalia was verbalizing. Ailani's contribution went unacknowledged and, as a consequence, Ailani went unrecognized as having achieved a greater level of generality than all the other student participants.

We see here that Ailani's semiotic means of objectification have increased in generality, suggesting increase in her mathematical sophistication and understanding. However, when we look closely at Ailani's remark to Thalia, "Bruh, it's like ten but you added one or two" (Line 155), we see that the mathematical basis of her assertion is uncertain. Ailani is not sure if one needs to add 1 or add 2 from the last known quantity (Fig. 4), so she adjusted the repeated summand to account for the difference between Thalia's answer of "fifteen" and her "nineteen." Despite the uncertainty Ailani is indicating here in Episode 5, and despite Ailani having admitted the possibility of a faulty counting strategy at the end of Episode 4, she nevertheless continued to hold on to this solution. She is personally invested in the task and will carry this solution through to the end. (The analysis of Episodes 8 & 11 show that her statement of "added one or two" comes back into play.)

Thalia responded to Ailani's assertions with a statement that reflects her first teeter into the Contextual mode of reasoning, at Line 154: "It does. It always adds—" but it was cut off by Ailani.

Both Ailani and Thalia made statements that express a mathematical procedure, but the illocutionary force of their utterances also positioned them as capable of asserting knowledge and thus established them as authority figures in the discussion. I claim that through these complex interactions, the threshold to operate in the Contextual mode was lowered because their status and identity were at stake. That is, the inherent power struggle of the conversation spurred them on to position themselves as authority figures within that dynamic, and what resulted were Ailani and Thalia's first Contextual statements.

With regard to Mr. Lam's role in the evolving dynamic. At Line 143, there was a notable shift in Mr. Lam's discourse from general discusive tactics (to encourage Xeni to participate) to operating in the Factual mode, when he highlighted certain aspects of the problem situation, marking them as imporant with his tone and cadence.

Mr. Lam revoiced Thalia's recursive generalization (Line 159), but he shifted the semiotic space from the Factual to the Contextual level of generality. So doing, Mr. Lam did not reiterate the additive/recursive component of Thalia's code, only that the first two entries remained constant while the variable was the third entry (Line 160: "It's staying two, three, and then something"). As Episode 5 came to an end, Mr. Lam walked away and tossed a final question to the team, asking them to consider cases beyond just the first ten figures (Line 160: "for any number?"). Thalia scoffs at Mr. Lam's final question, with one final statement in the Contextual mode that, for her, captures the complete solution (Line 161: You just keep adding the number—duhhh!"). Thalia may have interpreted the subtext of Mr. Lam's question as evaluating her solution as insufficient, and she disagreed with his assessment. Mr. Lam's question is a common tactic used in these kinds of instructional contexts involving figural patterns, intended for students to realize that recursive strategies are limited when dealing with cases much further down the line. Thalia's comments point to a mismatch between her perception of the requirements of the task and Mr. Lam's expectations.

Thalia's frustration ensues, all the way until Episode 11, where an overt conflict erupts between her, Mr. Lam, and Ailani. This conflict momentarily leaves the discursive frame of "mathematical practice" to a broader ideological frame. Below, I present a synopsis of Episode 11, so as to briefly highlight other aspects of the social-mathematical power dynamic. Specifically, the teacher makes two noteworthy moves in Episode 11 that have implications for the teaching and learning of mathematics more broadly. First, Mr. Lam challenges the group to generalize their Code to a normative algebraic one that can predict much larger numbers along the sequence. Second, Mr. Lam encourages Xeni to participate in the activity, to include her in the discourse. Yet so doing, whereas Mr. Lam's moves were pedagogically effective, they reifyied mathematical hierarchies that prioritize certain forms of argumentation over others (e.g., formal symbolism over explanations involving gesture and other communicative measures) and thus positioned Xeni with higher status.

Episode 11 (of 13) – Timestamp [25:18–29:54] – "My name is not 'Bruh'."

Mr. Lam returns to Team 1 and is met by Thalia with a question: "Are we done?" Mr. Lam confirms they are nearly done. Mr. Lam turns to Xeni and kneels down next to her to talk. He reminds both Ailani and Xeni that the task at hand involves working on the assigned pattern task, not drawing pictures. Thalia defends her team, exclaiming, "We just did it, look it! We just did that." Mr. Lam does not respond to Thalia and instead asks Xeni what she noticed about the Spiralateral pattern. Working together, Mr. Lam, Thalia, and Xeni co-construct a Code for the pattern. Xeni articulates her version of the final proposed solution using formal symbolism, stating that the last part of the Code is "n plus one." While Mr. Lam tries to unpack what Xeni means by "n," Thalia interjects and gives indication that she is growing frustrated with the conversation and is focused on task completion…which brings us to the opening scenario at the very top of this paper.

The solution that Team 1 has articulated, for items beyond Fig. 4, achieves the goal using a recursive technique that relies on known items along the sequence. Mr. Lam prompts the group to consider much larger numbers, such as Fig. 100, asking, "So we have to know the one before it [Fig. 100]? Is there a way that we don't have to know the one before it?" After a 2-second pause, Thalia stomps her feet and exclaims to Mr. Lam, "Bruh, why you asking all these questions?" Ailani stirs the conflict, interjecting loudly, "YES!" Mr. Lam calmly responds to Thalia: "My name is not 'Bruh.' My name is Mr. Lam, and I'm challenging you. That's why I'm asking you these questions, because I want you to get smarter." Thalia buries her face behind her hands and starts laughing sheepishly; her face still hidden behind her hands, she continues: "OK, Mr. Lam, why you asking (all these questions)."

Mr. Lam continues the mathematical conversation and challenges the group with "So could you even figure out Fig. 100 right now?" adding the parameter of "I don't tell you what Fig. 99 looks like, can you tell me Fig. 100?" Thalia explains her method for Fig.'s 1–10, but Mr. Lam insists that her method is insufficient for items much further along the sequence. Ailani leans back in her chair, throws her arms behind her, looks to Mr. Lam and leans forward as she exclaims: "Well maybe—OK so (like) two three and you don't have to know all of them, you know the first—you can know the original one and then (add on)! And then figure out Fig. 99." As she speaks, Ailani pounds the table with the marker in her hand and uses a tone of voice suggesting to Mr. Lam that she aims to "settle the score" once and for all in this confrontation. Thalia interrupts Ailani, softly clapping her hands together as she speaks, accentuating each word as she talks: "Why you talking so ratchet?" Mr. Lam weighs in with: "Don't get hyped up against me." As Mr. Lam is speaking, Thalia points to him and says: "Ratchet. Say it, don't get ratchet." Mr. Lam does not respond to Thalia's comment and instead steers the conversation back to the mathematics at hand, asking about Fig. *n*. The conversation continues for a few more minutes, until finally Xeni arrives at a closed solution of "n plus five." (See Gutiérrez, 2016, for richer description and deeper analysis of Episode 11, in which I explore a possible mechanism that links cognition to social structures and vice-versa in this episode.)

Conclusions

An analysis of mathematical ascendency, through the lens of the obj–subj dialectic, suggests that power dynamics and new mathematical understandings are co-constitutive through public discourse. The results of the study presented here, combined with findings from the larger project, show that the co-construction of power relations and the co-construction of knowledge are not necessarily distinct processes. Surely each of these processes can be independently instantiated in practice, but I maintain that they also become co-constitutive at certain points in the discourse, thus the learning–power imbrication emerges.

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