# Fostering University Freshmen’s Mathematical Argumentation Skills With Collaboration Scripts

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**Abstract:** Students often have problems formulating und using arguments in mathematical contexts. Therefore, we investigated to what extent a collaboration script helps students overcome their problems and acquire mathematical argumentation skills. In two previous studies, we showed that collaboration scripts can have positive effects on learning cross-domain argumentation skills in the mathematical context. Yet, the effectiveness of the script depended on individual prerequisites such as final high school grade (GPA) and self-regulation skills. In this study, $N = 96$ participants learned in one of three script conditions. We found that a high-structured domain-general collaboration script for argumentation was more effective for acquiring domain-specific mathematical argumentation skills than a low-structured or an adaptable one. Furthermore, only in the condition with the low-structured script, learners’ self-regulated learning skills played an important role for the learning outcomes.

**Keywords**: mathematics, argumentation, collaboration scripts

## Mathematical argumentation

Argumentation is a common mode for knowledge-generating dialogues in social science, politics, or everyday negotiation. Against this background, investigating the potentials of argumentation for knowledge-generation in a mathematical context might not be that intuitive. Yet, argumentation also plays an important role in mathematics, both in informal mathematical reasoning (Thurston, 1994) and even more so when it comes to formal proof, which is considered the ultimate form of mathematical argument. Finding promising conjectures and constructing mathematical proofs typically requires the use of heuristic strategies (rather than algorithms) and frequent comparisons between the promises and problems of different solution strategies (Reiss et al., 2008). Moreover, the construction of an acceptable proof requires the repeated production and evaluation of single arguments as well as of the structure of the overall argumentation. This process of proof construction, which is “usually held informally between mathematicians to develop, discuss or communicate mathematical problems and results” (Douek, 1999, p. 129; see also Thurston, 1994), can be considered a substantial argumentation process, requiring justifications of the strategic decisions as well as the mathematical inferences made. Thus, during the process of finding a mathematical conjecture or proof, argumentation skills are required at different points either to construct sound arguments, e.g. with the components introduced by Toulmin’s (1958) model of argument (Pease, Smaill, Colton, & Lee, 2009), or to be able to engage in an argumentative dialogue.

According to an experts’ process model proposed by Boero (1999), this process comprises phases of exploration as well as phases of consolidation of arguments into a deductive chain. The explorative phases are executed in a rather tentative way that includes revisions of steps made before. The discussion with others during those phases seems to be a promising way to frequently evaluate and strengthen the chosen approach to solve the proof task, since the others could bring new ideas and perspectives to refine ideas and arguments. Other phases include the selection and organization of coherent arguments which ideally lead to the formal proof requested by the initial task (for elaborated descriptions of the process of proof see Boero, 1999; Reiss, et al., 2008), which require evaluation of the arguments according to standards of the discipline. Also this evaluation process can benefit from monitoring by collaboration partners.

Apart from supporting problem solving processes, the engagement in an argumentative dialogue has more benefits to offer. The process of constructing and applying arguments in a social discursive collaborative learning process is ascribed a high potential for deeper elaboration of the learning content and thus a key moderator for the enhancement of learning processes (“arguing to learn”; Andriessen, Baker, & Suthers, 2003). Even though the formal rules for accepting proofs are quite specific to mathematics, general guidelines for the construction of arguments in a collaborative discussion as well as for a sustainable social discursive process of argumentation between dialogue partners are considered to be more or less applicable to other domains as well. For instance, Toulmin’s (1958) argument model is widely used for the evaluation of single arguments in multiple domains (van

Eemeren & Grootendorst, 2004). Also the dialectical format of argumentation - simplified as the cycle of 'argument', 'counter-argument', and 'synthesis' - might function as a common ground for social discursive activities where two or more dialogue partner are engaged in an argumentative discourse (Leitão, 2000). Studies have detected that students often struggle when trying to construct valid mathematical argumentations (e.g. Heinze, Reiss, & Rudolph, 2005). Students are also often not able to engage in a deeper elaborative dialogue while learning collaboratively (Cohen, 1994). As students have problems effectively using argumentation in mathematical learning processes (Hillbert et al., 2008), it is important to find ways to appropriately scaffold students' argumentation in the context of mathematical proof tasks.

## Scaffolding mathematical argumentation

One way to help students to acquire knowledge and skills related to mathematical argumentation is by offering collaboration scripts for argumentation. Collaboration scripts aim at guiding students through meaningful and beneficial collaborative learning processes by distributing roles and activities among the students (King, 2007; Kollar, Fischer, & Hesse, 2006). Collaboration scripts have shown to be effective in both face-to-face and computer-supported collaborative learning (e.g. O'Donnell & Dansereau, 1992; Rummel & Spada, 2005, Weinberger, Fischer, & Mandl, 2003). For the design of collaboration scripts, an explicit reference toargumentation frameworks proved to be helpful. For example, Weinberger, Stegmann and Fischer (2010) developed a script that guided students through an argumentative discourse by offering explicit support on crucial points in the argumentation process. For instance, when students were supposed to construct arguments or counterarguments, they were asked to address single elements of Toulmin's (1958) argument model (e.g. claim, data, qualifier). In other computer-supported collaboration scripts, students' discourses were guided by distributing the roles of discussants (e.g. De Wever, Schellens Van Keer, & Valcke, 2008) or by prompting them to perform single activities of a discursive argumentation cycle (e.g. Hron, Hesse, Cress, & Giovis, 2000).

In recent studies, the collaboration script approach has been transferred to the mathematical domain as well. In an own study, an argumentation-related collaboration script showed positive effects on students' cross-domain argumentation skills (Kollar et al., 2014), both in the learning process and a subsequent individual post test.However, positive effects on domain-specific mathematical argumentation skills could not be found. Additionally, in this study the final high school qualification grade (GPA) was significantly related to the effectiveness of the collaboration script on domain-general argumentation skills. The better the GPA grade, the more the students benefited from the collaboration script (Kollar et al.,, 2014). Yet, in other domains, collaboration scripts were found to exert positive effects on domain-specific learning (e.g. Weinberger, et al., 2010), albeit not consistently. As the GPA was such an important factor for the effectiveness of the collaboration script in the study by Kollar et al. it can be hypothesized that the used collaboration script did not offer enough hints and structure, especially for learners with lower GPA.

A further criticism that is often raised against the use of collaboration scripts to support collaborative learning efforts is that collaboration scripts may structure learners' activities in a too rigid way. This might lead to "overscripting", i.e. that the collaboration script inhibits learning by constraining learners in using their own, possibly effective learning strategies (Dillenbourg, 2002). One way to avoid overscripting is to offer the learners adaptive collaboration scripts. These collaboration scripts would automatically be adapted to a learner's current needs (e.g. Diziol, Walker, Rummel, & Koedinger, 2010). Unfortunately, such adaptive learning scenarios are enormously complex and expensive to develop. Therefore, a more feasible way to avoid overscripting might be to create adaptable collaboration scripts that give the learners themselves the possibility to adapt the script to their own needs. Yet, the learners might need a certain amount of self-regulation skills to benefit from such adaptable learning scenarios (Vogel et al., 2015).

## Goals of the study

The goal of this study was to compare the effects of a high-structured collaboration script, a low-structured collaboration script and an adaptable collaboration script in the context of mathematical proof tasks on students' acquisition of mathematical argumentation skills. Furthermore, we investigated to what extent the learner's GPA and their self-regulated learning skills would be related to learning in each of the three collaboration scripts. In line with prior research, we expected the high-structured collaboration script to have a more positive impact on learning mathematical argumentation skills than the low-structured collaboration script. Furthermore, we also expected students in the adaptable script condition to outperform students who received the low-structured collaboration script, as the adaptable script offers each learner the amount of structure s/he (thinks s/he) needs. Therefore, we expected that the self-regulated learning skills would be positively related to learning especially in the condition with the adaptable script, but also in the low-structured condition since both offer learners the opportunity to self-regulate their learning much more strongly than the high-structured script. We expected the GPA to be positively related to learning in the low-structured script condition.

## Research questions

RQ1: To what extent does a low-structured collaboration script, a high structured collaboration script and an adaptable collaboration script affect learner's acquisition of mathematical argumentation skills?

RQ2: To what extent are the learners' self-regulated learning skills related to learning mathematical argumentation skills when learning with either a low-structured, a high-structured or an adaptable collaboration script?

RQ3: To what extent is the learners' GPA related to learning mathematical argumentation skills when learning with either a low-structured, a high-structured or an adaptable collaboration script?

## Methods

### Participants and design

The study was implemented in a two weeks preparation course that was conducted at three different German universities in fall 2011. Participants were *N* = 96 (*n* = 58 female; *n* = 38 male) mathematics students and mathematics teacher students at the beginning of their university studies. Their mean age was *M* = 19.53 years. The students were randomly assigned to the three treatment conditions (1) low-structured argumentation script (*n* = 29), (2) high-structured argumentation script (*n* = 34) and (3) adaptable argumentation script (*n* = 33).

In each condition, we grouped students in dyads of high as well as low achievers, based on their final high school qualification grade. For each of the three learning tasks, we randomly formed new dyads within the high and low achievers groups nested in the experimental conditions to minimize the impact on knowledge acquisition one specific learning partner might exert on a given learner.

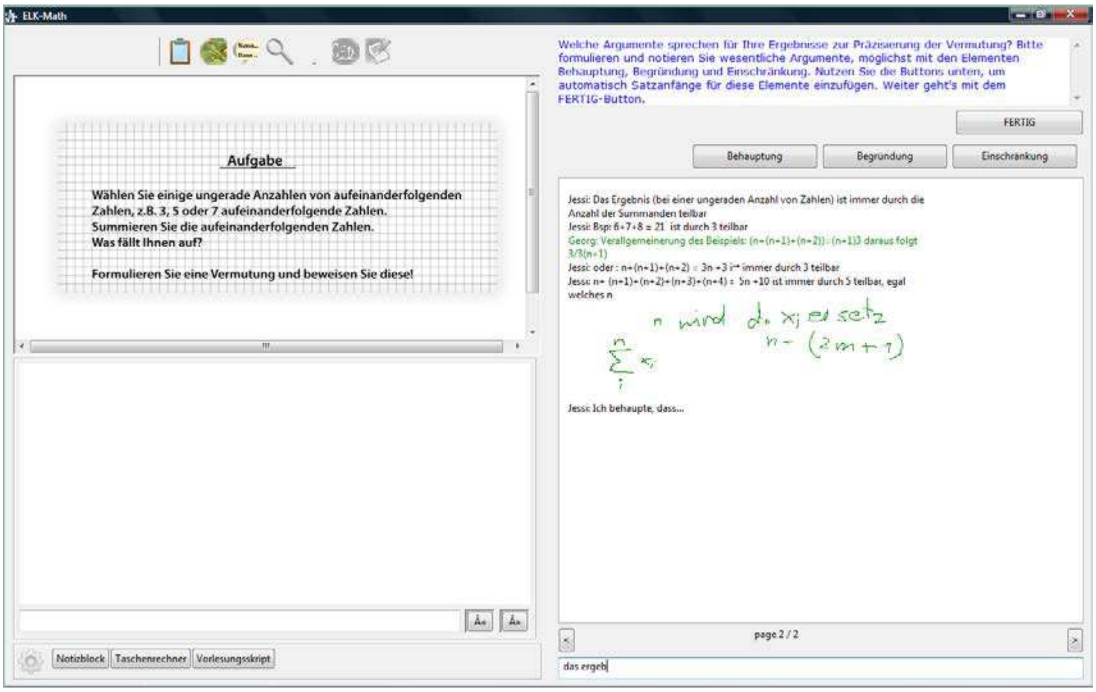


Figure 1. Screenshot of the computer-supported learning environment with the proof problem on the left side, script prompts on the upper right side and the communication area below the script prompts on the right side.

### Setting and learning environment

Students learned collaboratively in dyads in a computer-supported learning environment (see Figure 1) on three different problem solving tasks in the context of mathematical proof. Additionally to the proof tasks, the learning environment offered six driving questions derived from Boero's steps of mathematical proof (Nadolski, Kirschner,

& van Merriënboer, 2006). The learners were seated vis-à-vis each other and each learner was equipped with one laptop, a graphic tablet and a mouse. The two learners of each learning dyad were connected via a computer-supported learning environment that displayed the current mathematical proof task and the script prompts in the respective treatment conditions. The computer-supported learning environment provided the learning task itself and a shared communication area where the two learning partners could exchange text messages as well as drawings. They were also asked to type the most important results of their discussion into the communication area even though they were also allowed to discuss orally.

## Conditions of the learning environment and learners' pre-requisites

*Type of collaboration script.* Three types of collaboration scripts were compared in this study: a low-structured script, a high-structured script and an adaptable script. The students were asked to collaboratively discuss their ideas about the proof tasks in all treatment conditions. In the condition with the low-structured script, prompts repeatedly sequenced the discussion into the phases (1) argument, (2) counterargument and (3) synthesis. During each phase, prompts distributed the roles of 'talker' and 'listener' to the learning partners. The roles were switched for every new argument cycle. . In the condition with the high-structured script, students were additionally prompted to formulate sound arguments (i.e. to formulate claims, data and rebuttals based on Toulmin's, 1958 argumentation model) in each of these three steps. In the condition with the adaptable argumentation script, learners were allowed to choose between the high structured and the low structured argumentation script at six points in time within each treatment session (see Figure 2). Before they adapted the script, the learners had to discuss which script they wanted to take in order to come to a joint solution.

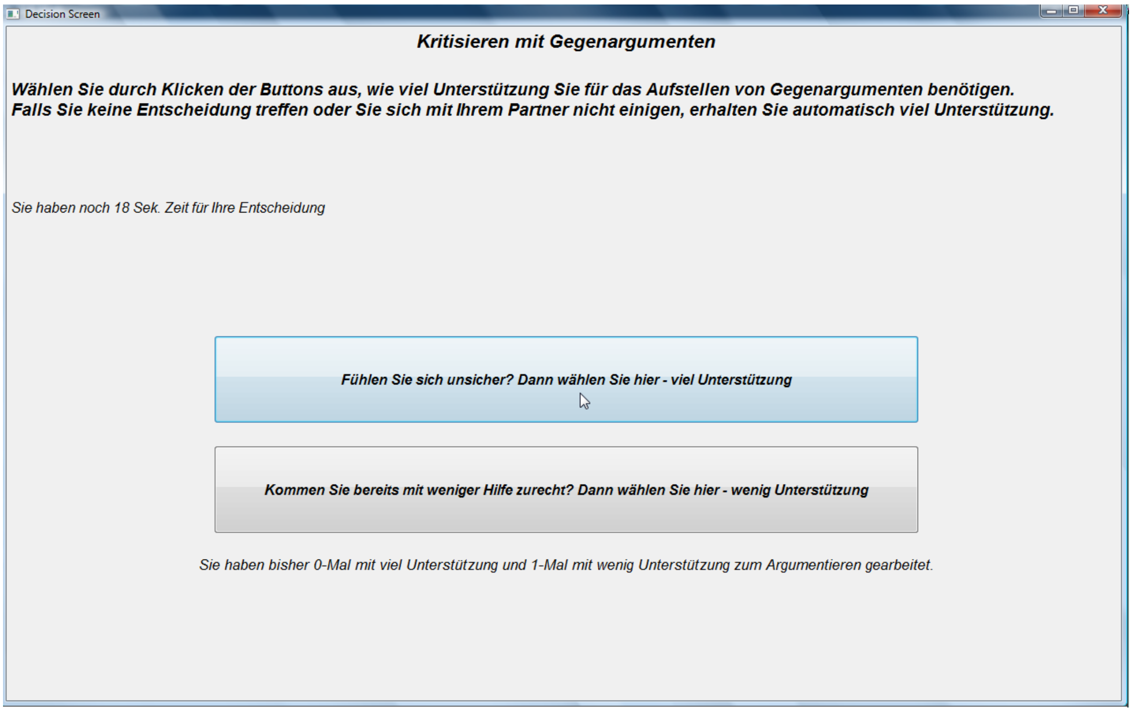


Figure 2. Screenshot of the decision screen with two buttons to adapt the script either into a high-structured or into a low-structured script.

*GPA.* For the GPA as moderator variable the students were asked for their final high school qualification grade. The grades ranged from 1.00 to 3.50 with an average grade of 1.94.

*Self-regulation skills*. Self-regulation skills were measured with an 18-items questionnaire (adapted from Fisher, King, & Tague, 2001) in which students rated the extent to which they typically apply certain self-regulation strategies (e.g.; "I prefer to plan my own learning"; "I am systematic in my learning"). The resulting scale proved reliable (Cronbach's $\alpha = .77$).

## Instruments and outcome measures

To analyze students' individual learning outcomes, we conducted parallel pre- and post-tests one day before and after the treatment phases, comprising 17 open items each (cf. Kollar et al., 2014). Five items on schematic argumentation with elementary rules from number theory (e.g., "Show that for all natural numbers, a and b, the following statement is true: If 7 divides a+3b then 7 divides 2a+13b."), which required transformations of the algebraic expression and application of rules from the courses' number theory lectures. Proof skills in elementary number theory were examined by six items (e.g., "Prove the following statement: The sum of five consecutive numbers is divisible by five."), and six items tested performance in open-ended argumentation problems (e.g., "Prove or refute the following statement for natural numbers a and b: If you multiply the sum of a and b with the difference of a and b, you will always obtain an even number."). Two trained, independent raters coded all items. Inter-rater reliability was good (Mean of $ICC_{unjust}$ = .79). Where discrepancies remained, raters discussed them until they reached a consensus. Reliability was good for both tests (Cronbach's alpha: $\alpha$ = .82 for the pre-test, $\alpha$ = .80 for the post-test). For the statistical analyses all scores were merged and scaled to values between zero (nothing correct at all) and one (everything correct).

## Results

First, we checked if the three script conditions differed in the pre-test mathematical argumentation skills of the participants. An ANOVA did not yield significant differences ($F(2,93) = 2.10$, *n.s.*). To answer the first research question, an ANCOVA with pre-test mathematical argumentation skills as a covariate was conducted.

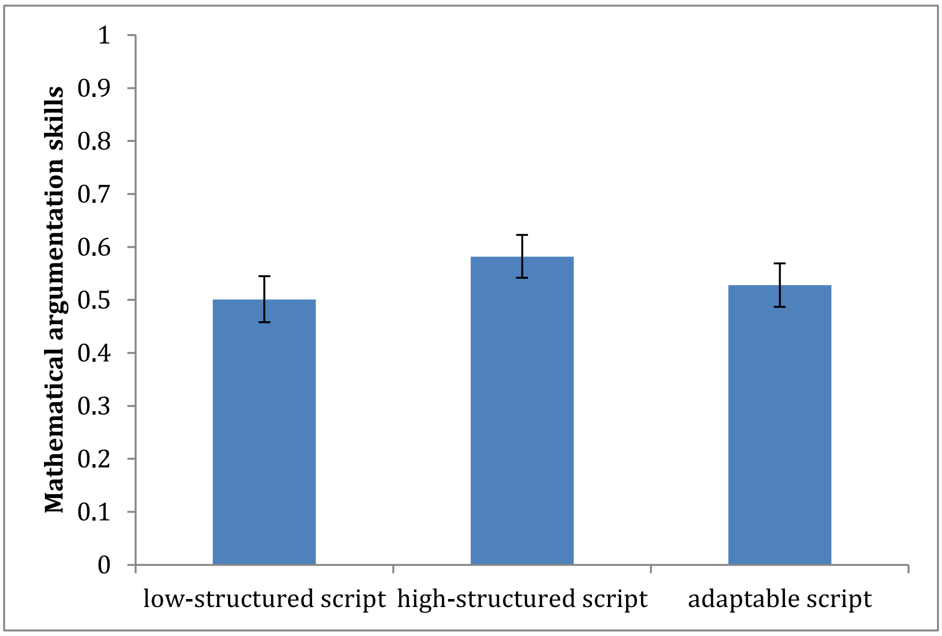


Figure 2. Learners' mathematical argumentation skills by script condition.

The descriptive statistics showed that regarding the outcome score, learners in the high-structured script condition outperformed the learners in the adaptable script condition. Learners in the low-structured script condition reached the lowest outcome score for mathematical argumentation skills. The ANCOVA revealed a significant effect of the script conditions on students' acquisition of mathematical argumentation skills ($F(2,92) = 3.91$, $p = .02$, part. $\eta^2 = .08$). Post-hoc comparisons between the three levels of the independent variable showed that it was the condition with the high-structured script ($M = 0.58$, $SE = 0.02$) that reached significantly higher results in mathematical argumentation skills than the condition with the low-structured script ($M = 0.50$, $SE = 0.02$). The adaptable script condition ($M = 0.53$, $SE = 0.02$) was not significantly different from the high- and low-structured script (see Figure 2).

To answer research questions RQ2 and RQ3, we conducted regression analyses in each of the three conditions with pre-test mathematical argumentation skills, self-regulation skills and GPA as predictors and post-test mathematical argumentation skills as dependent variable. Results of the regression analyses showed that only in the low-structured script condition, self-regulation skills were significantly and positively related to the acquisition of mathematical argumentation skills. GPA was no significant predictor at all for post-test mathematical argumentation skills (see Table 1). Also, in all conditions the pre-test mathematical argumentation skills served as significant predictor for the post-test argumentation skills.

Table 1: Regression models for learning mathematical argumentation skills within each of the three conditions.

| | *B* | *SE B* | β |
|---|---|---|---|
| **low-structured script[1]** | | | |
| Pre-test mathematical argumentation skills | 0.708 | 0.116 | .738*** |
| Self-regulation skills | 0.109 | 0.051 | .264* |
| GPA | 0.049 | 0.038 | .202 |
| **high-structured script[2]** | | | |
| Pre-test mathematical argumentation skills | 0.633 | 0.109 | .742*** |
| Self-regulation skills | 0.002 | 0.056 | .005 |
| GPA | 0.033 | 0.041 | .100 |
| **adaptable script[3]** | | | |
| Pre-test mathematical argumentation skills | 0.609 | 0.130 | .686*** |
| Self-regulation skills | 0.047 | 0.054 | .110 |
| GPA | 0.019 | 0.038 | .079 |

Notes: [1]$R^2 = .662$***, [2]$R^2 = .591$***, [3]$R^2 = .597$***; *$p < .05$, **$p < .01$, ***$p < .001$

## Discussion and conclusions

The results on mathematical argumentation skills are in line with our hypothesis reflecting the results of studies on collaboration scripts for argumentation within other domains (e.g. Weinberger et al., 2010). Thus, learning with a high structured collaboration script helped students extend their domain-specific skills in mathematical argumentation. Since it was the high-structured script that led to better skill acquisition compared to the low-structured script, the findings of this study are not favouring a strategy of minimal scripting when trying to avoid possible overscripting (Dillenbourg, 2002). Furthermore, the difference between the high structured and the low structured script lies in the additional prompting for the use of Toulmin's (1958) argument structure. The use of the structure itself might have helped student to elaborate more deeply on the mathematical content and through this they may have acquired better mathematical argumentation skills (King, 2007). It is plausible that increasingly making the script more complex would eventually lead to an overscripting (Dillenbourg, 2002). The more important question than *how much* script support is necessary, might be *which script support* is actually helpful. Thus, a systematic variation of different script components based on different frameworks (Toulmin, 1958, Leitao, 2002) might be a promising research direction for the future.

As Vogel et al. (2014) point out, collaboration scripts rarely lead to domain-specific learning gain, if they are not combined with adequate domain-specific instructional support. In a past study by Kollar et al. (2014), applying a collaboration script did not support the acquisition of mathematical argumentation skills substantially better than unstructured collaboration, when combined with problem solving or heuristic worked examples. The guided problem solving support using driving questions (Nadolski et al., 2006) based on an experts' model of the proof process (Boero, 1999), as applied in this study, represents a compromise between the two approaches used by Kollar et al. (2014). The results indicate that this approach might offer better opportunities for students to actually make use of the collaboration script support.

The support with an adaptable collaboration script for argumentation was not significantly different from leaning with a high-structured or a low-structured script regarding the acquisition of mathematical argumentation skills. Maybe the learners were overwhelmed with the task to adapt the collaboration script to their own needs. Further studies should investigate which kind of support would be necessary for students for being able to adapt the collaboration script in a fruitful way. Also, computer support might be used to adapt the script automatically to the responses students give (e.g. Diziol, Walker, Rummel, & Koedinger, 2010).

Furthermore, learners in the condition with the low-structured collaboration script reached higher scores in the final mathematical argumentation skills test when they had good self-regulation skills. This implies that especially in the low-structured script condition, the learners with better self-regulation skills might have been able to adjust their argumentation on a higher level than it was supported by the script and thus engage in more elaborated learning dialogue, finally achieving better learning outcomes (King, 2007).

The same effect could not be found in the other two conditions. This is especially surprising for the adaptable script condition. In this condition the learners were explicitly asked to adapt the learning environment to their own needs. We expected that an adequate adaption of the learning environment would require a certain amount of self-regulation skills. Yet, there was no difference for students in the adaptable script condition with higher or lower self-regulation skill in their acquisition of mathematical argumentation skills. One reason for this could be that all learners, no matter if they were good self-regulators or not, always adapted the script into one direction without reflecting if this script might fit to their needs. Again, this is a further argument to find better instructions to help learners within the process of adapting their learning environments.

What can be seen as success for the design of the collaboration scripts is the fact that in none of the three conditions the GPA was an important factor for learning mathematical argumentation skills. The collaboration script seems to "balance" to some extent the differences learners might have which might be expressed by the GPA the learners have.

Of course, this study is not without limitations. What can be seen as drawback of this study is that it has no real control condition in the sense of a waiting group or a group learning without any support. Yet, this study was deliberately designed to compare different kinds of collaboration scripts which should lead to better insights about how collaboration scripts should be designed in order to support domain-specific learning.

In conclusion, more structure seems to be better than less when we aim at designing collaboration scripts to positively affect domain-specific learning outcomes. Mainly this might be true because less structure is accompanied by higher demands on self-regulation skills which are quite unevenly distributed among secondary school students as well as freshmen at university.

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