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Exploring the Interplay of Mathematical Values in Algebra Problem-Solving

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Abstract: Mathematical problem-solving is a value-laden activity, yet how students' mathematical values shape their problem-solving remains unclear. This mixed-methods study examined the relationship between 338 high school students' mathematical values and their procedural flexibility in algebra. Quantitative survey data revealed that while valuing Rationalism–Objectivism predicted greater flexibility, this relationship was moderated by Control–Progress values (β = -0.056, p = .062), with the benefit strongest for students holding moderate Control orientations. Qualitative interviews with four focal students suggested that the most flexible problem-solvers were those with a more balanced Control–Progress value, who adapted strategies to context rather than adhering to rigid computational routines. Findings suggest that procedural flexibility may emerge from the interplay between valuing logic and representations and maintaining a more moderate stance on mastery. Thus, fostering flexibility may require pedagogical approaches that help students develop balanced mathematical values.

Introduction

A persistent challenge in algebra instruction is that many students rely on single routine procedures even when multiple solution strategies are available. This lack of procedural flexibility limits students' ability to adapt their reasoning to new problem contexts and has been identified as a key obstacle to deeper mathematical understanding (Star, 2005). Although prior interventions have attempted to improve flexibility through curriculum design and instructional practices (e.g., Durkin et al., 2023; Wang et al., in press), these approaches have produced uneven results. One potential explanation is that they focus primarily on what students know and can do, while paying less attention to what students value in mathematical activity (Wang and Star, 2023).

Mathematics is not only a system of knowledge but also a human endeavor shaped by cultural and epistemological values (Kline, 1953; Schoenfeld, 2020). These values influence what counts as a situationally appropriate strategy and how students engage with mathematical tasks (Nakajima, 1981; Yamazaki, 2024). Bishop (1988) emphasized that mathematics is a value-laden subject and identified three pairs of mathematical values: Rationalism–Objectivism, Openness–Mystery, Control–Progress. Because values are inherently conative (Seah, 2019), they can shape students' actions in problem-solving situations (Wang, 2025). Thus, mathematical values provide a lens for understanding why students may approach the same task in different ways.

Recent studies suggest a potential connection between values and mathematical problem-solving. On one hand, values are embedded in the design of mathematical tasks and in the ways these tasks are used for instruction, shaping the kinds of thinking and learning they encouraged. For example, Dede et al. (2021) showed that mathematical modeling tasks embody multiple value dimensions, and Kalogeropoulos et al. (2024) highlighted the importance of aligning instruction with students' value orientations. On the other hand, students' interactions with tasks also reveal underlying values. Studies have shown that different types of tasks elicit different value patterns (e.g., Baba et al., 2024; Safura et al., 2018), and that students often prioritize instrumental approaches emphasizing efficiency over conceptual understanding (Seah, 1999; Davis et al., 2024).

Despite these advances, little is known about how mathematical values relate specifically to procedural flexibility in algebra problem-solving. This topic is pedagogically significant. If students' strategic choices reflect not only their knowledge but also their value orientations, then efforts to foster flexibility may require instructional approaches that explicitly engage with and shape these values. Understanding this relationship could, therefore, inform the design of learning environments that support more adaptive and flexible problem-solving. Accordingly, we ask two research questions. First, how do the three pairs of mathematical values relate to students' procedural flexibility in algebra problem-solving? Second, what are the similarities and differences in approaching algebra problem-solving tasks among students with different values? Here, procedural flexibility is defined as the ability to solve a mathematical problem with multiple strategies and being able to select the most situationally appropriate strategy for the given problem context (e.g., Star, 2005; Wang & Star, 2023). By identifying patterns in high

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school students' value preferences and their associated problem-solving behaviors, this study aims to inform value-responsive instruction and contribute empirical insight into how values can be leveraged to support flexible algebra problem-solving.

Theoretical framework

The current paper investigates the three pairs of mathematical values as defined in Bishop (1988). For the first pair, Rationalism emphasizes reasoning or logical thinking; it dominates behaviors like explaining, arguing, showing, and using mathematical proof. Objectivism prefers materializing contexts; for example, we can use mathematical objects, such as diagrams and symbols, to concretize mathematical ideas. For the second pair, Openness values more about the demonstration of facts in presenting and defending mathematical ideas, while Mystery values more about wonder, fascination, and the mystique of mathematical ideas. For the third pair, Control foregrounds a mastery of rules, facts, procedures, and established criteria; Progress highlights the ways mathematical ideas grow and develop through alternative theories.

Instead of perceiving one type of value as more important than others, we agree with Bishop (1988) that students need to appreciate these values in a balanced way. We hypothesized that students with more balanced value pairings would show greater procedural flexibility in algebra tasks. We also hypothesized that the relationship between Rationalism–Objectivism and flexibility would vary by students' orientations toward Control and Progress. Through a close reading of how value interplay relates to problem-solving behavior, our goal is to understand how deeply affective factors like values influence students' mathematical learning.

Methods

We employed a concurrent complementary mixed-methods design (Caracelli & Greene, 1993) to investigate the relationships between pairs of mathematical values and students' algebra problem-solving. In Phase 1, we conducted quantitative analysis on the five-point bipolar Likert-scale items selected from Section B of the What I Find Important (WIFI; Seah, Baba, & Zhang, 2017) Survey to identify patterns in our sample. We estimated separate multilevel models for procedural flexibility, controlling for age, gender, and ethnicity, and specifying a random-effects structure of $(1 \mid Teacher\ /\ Class)$ to account for students nested within classes and teachers. Empty surveys were excluded from analysis ($\approx$ 1.2 % of cases). In Phase 2, we conducted qualitative analysis of think-aloud interview recordings of students working on algebra tasks. These interviews provided evidence of students' problem-solving behaviors and were used to examine how their value preferences were reflected in their strategic choices and explanations during problem-solving. The qualitative data were used to complement, illustrate, and elaborate on the quantitative findings from Phase 1 rather than to establish causal relationships.

This study is part of a larger research project focusing on improving Chinese students' mathematical problem-solving skills. For this study, to rule out potential intervention effects, we used survey data from the pretest administered at the beginning of the semester. This study was conducted in an urban private high school located in southwestern China. A total of 338 Grade 10 students participated in the study. All students were of Chinese background, with 95% identifying as Han ethnicity. The student body primarily represented middle socioeconomic status. Gender was relatively balanced (girls 47.3%; boys 52.7%) across participating classes, and students entered the school from a variety of local middle schools with prior math scores of (Mean = 109.3 out of 150, SD = 40). The participating students were drawn from twelve classes. While the sampled students were in the same school at the time of study, they attended different middle schools (Grades 7 to 9) and primary schools (Grades 1 to 6). Under a broadly similar cultural context (southwestern China), their values may nevertheless have been shaped by different school environments and classroom experiences, which provided variation for interpreting patterns of values and problem-solving skills in the sample.

Results

Quantitative findings

To contextualize the regression models, we first examined the overall distribution of value orientations in the sample ($N = 334$). Figure 1 presents a violin plot of the three bipolar value pairs, Rationalism–Objectivism (Value 9), Openness–Mystery (Value 10), and Control–Progress (Value 11). The distributions were relatively balanced around the midpoint, with a slight tendency for the sample to lean toward Objectivism and Control. This pattern suggests that, as a group, students placed a somewhat higher value on concrete representation and procedural mastery than on logical reasoning or exploration perspectives.

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Figure 1
Distributions of Mathematical Value Preferences

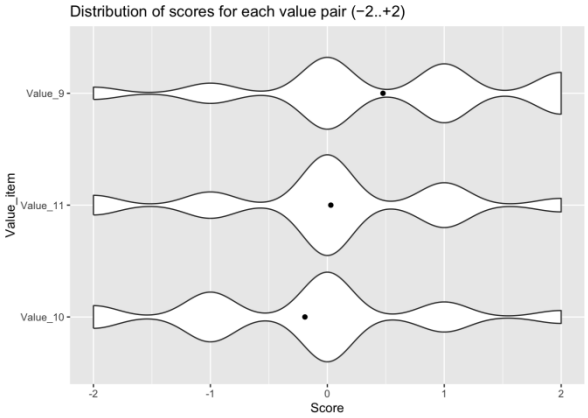
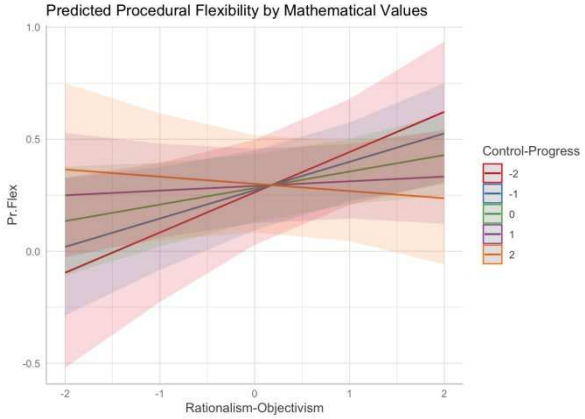


Figure 2
Predicted Procedural Flexibility Scores



The multilevel regression model predicting procedural flexibility revealed a small but statistically significant main effect of Rationalism–Objectivism ($\beta = 0.0767, SE = 0.039, t = 1.97, p = .050$). The Rationalism–Objectivism × Control–Progress interaction was marginally significant ($\beta = -0.056, SE = 0.030, t = -1.88, p = .062$). No other predictors were statistically significant (all $p > .18$). Random effects indicated minimal between-teacher variance ($class\ SD = 0.14, teacher \approx 0, residual\ SD = 0.73$). The positive main effect indicates that students with a stronger orientation toward Rationalism demonstrated greater procedural flexibility. However, this association was conditional on students' Control–Progress orientation, as indicated by the interaction term.

As illustrated in the predicted values plot (Figure 2), the positive association between Rationalism–Objectivism and flexibility was most pronounced for students with low to moderate Control orientation. In this group, greater emphasis on logical reasoning was associated with higher procedural flexibility. In contrast, for students with a strong Progress orientation, this association was no longer evident, as indicated by a near-flat slope. This suggests that valuing logical reasoning and representations does not uniformly translate into flexibility; rather, its effect depends on how students position themselves with respect to procedural mastery versus idea development. Aligning with our hypothesis, the models together indicate that procedural flexibility in algebra problem-solving is not driven by a single value in isolation but by the interaction between value orientations, particularly the interplay between Rationalism–Objectivism and Control–Progress.

Qualitative findings

To contextualize the quantitative pattern, we conducted a thematic analysis using a primarily deductive coding approach informed by Bishop's (1988) value framework of four focal students (Gabriel, Cameron, Hayden, and Francis; all pseudonyms). Cases were selected to represent variation across value dimensions, and analysis involved iterative coding of interview transcripts, followed by cross-case comparison to identify patterns linking value orientations and problem-solving behaviors. Analysis of the interviews revealed that the use of concrete representations was common, and how this connects to procedural flexibility was critically shaped by the students' value preferences toward Control or Progress.

Two students, characterized by strong Control values, demonstrated how a focus on mastering rules and procedures appeared to channel their logical reasoning toward singular and efficient solutions. Gabriel articulated an approach centered on systematic practice and categorization, stating the need for "summarizing problem types systematically." Their Control value preference was expressed through a preference for methods with "fewer steps and less thinking," and he explicitly prioritized mastery of a single reliable procedure, arguing that "doing one method well is better than trying too many." Similarly, Cameron favored formula-centered solutions; in their opinion, mathematical formulas provide an objective procedure. They prefer solution strategies with operational simplicity, noting that they were "easier to understand or operate, and the calculation is simpler." For both students, a strong Control value appeared to channel their logical reasoning toward identifying and executing the most straightforward and low-risk path, which was associated with more limited procedural exploration.

In contrast, Hayden, with a more balanced Control–Progress value preference, demonstrated how a moderate value stance could foster high procedural flexibility. His Rationalism was evident in his emphasis on foundational understanding as a prerequisite for strategic choice: "You first need to understand the knowledge point; only then can you make sense of the problem and come up with a solution." He did not seek a single best

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method, but rather the most fitting one, explaining that identifying "which part of the problem provides the breakthrough" was key to selecting "the method that fits it best." This indicates that his use of concrete representations and logical reasoning analysis were dynamically adapted to the problem context. His procedural flexibility appeared to be supported by a more balanced Control–Progress stance, which allowed him to adjust his reasoning to the demands of the problem rather than commit to a single preferred method.

A different dynamic was observed for Francis, whose strong Progress value preference appeared to redirect his Rationalism toward conceptual generalization and theory-building. He deeply valued understanding definitions and properties, stating the need to "understand both specific and basic applications." His balanced Rationalism – Objectivism value was shown in his ability to fluidly translate between abstract symbols and concrete images. His Progress value was characterized by seeing overarching patterns in the problem exploration, as when he immediately connected the condition $x > y > 0$ to the "arithmetic–geometric mean inequality." He further generalized that "different-looking problems often share the same underlying principle," illustrating that his procedural flexibility was not expressed through a fixated singular value but through a mix of conceptual transfers and idea expansions, hallmarks of a Progress value.

Aligning with our hypothesis, these cases suggest that the Rationalism–Objectivism pair does not operate in isolation. With a strong Control orientation, it was associated with problem-solving that was robust but less flexible, with greater emphasis on procedural mastery and efficiency. When moderated by a balanced Control–Progress stance, it was associated with more responsive, context-sensitive strategy use. When aligned with a strong Progress orientation, Rationalism–Objectivism fuels a form of procedural flexibility aimed at identifying unifying principles and developmental connections between ideas. They suggest that procedural flexibility in algebra problem-solving is not simply a product of valuing logic and representations but may depend on how these values are tempered by a student's stance on mastery versus the growth of knowledge.

Discussion

This study investigated the relationship between students' mathematical values and their procedural flexibility in algebra. Integrating quantitative and qualitative findings reveals that flexibility is not determined by any single value but emerges from a specific interplay between them. The positive influence of valuing logic and representations on flexibility is contingent upon a student's stance on procedural mastery and idea exploration. This synergy suggests that flexibility arises when values of representation and procedure are more balanced. A strong Control orientation appeared to channel the cognitive benefits of Rationalism–Objectivism toward procedural efficiency and computational coherence, which was associated with robust but relatively rigid problem-solving. Conversely, a strong Progress orientation appeared to direct these same benefits toward conceptual generalization, prioritizing theoretical connections over procedural variation. In both cases, the potential for flexible strategy use is constrained. The most adaptable students in our qualitative cases were those with a more moderate profile, who could leverage their logical reasoning to dynamically navigate a problem's context rather than merely execute a routine or identify an abstract principle.

These findings refine our understanding of problem-solving by showing that lower flexibility need not stem from a deficit in ability but can reflect a coherent value system that intentionally prioritizes mastery or theoretical insight over exploration. This perspective raises the possibility that students' underlying value systems may be one factor contributing to the mixed results observed in prior instructional interventions targeting flexibility. Consequently, instructional efforts to foster flexibility may be more effective if they attend to students' underlying values as well as their procedural knowledge. Pedagogy could aim to cultivate a more balanced value stance by framing computational procedures as tools for sense-making rather than prescribed rules, and by linking conceptual development to its utility in enabling diverse procedural pathways. For example, instruction might invite students to compare multiple solution methods, explain why one method fits a particular problem better than another, and treat procedures as resources for reasoning rather than as steps to be followed mechanically. Future research could explore whether and how other conative factors, such as self-regulation skills, mediate the relationships between these values and student problem-solving behaviors.

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