International Society of the Learning Sciences

Symbol Grounding and Re-Grounding in Balance and Movement: Insights From Children's Mathematical Compositions

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Abstract: In teaching mathematics, educators often seek to address the "symbol grounding problem"– the challenge of giving meaning to symbolic expressions. One avenue for addressing this challenge involves coordinating symbolic expressions with bodily experiences. Extending and integrating prior work on embodied design for symbol grounding and on balance metaphors in the teaching of algebra, we explore grounding math concepts in the bodily sense of balance, focusing specifically on the additive inverse principle. We conducted 9 semi-structured clinical task-based interviews with a developing design, the Balance Number Line, in which children use arithmetic symbolic expressions and a physical number line to compose and convey a set of bodily rocking movements to other children. We used multimodal video analysis of coordination across semiotic media (including language, gesture, arithmetic symbolic expressions, the number line, and hand and body movement) to examine whether and how children grounded arithmetic symbolic expressions in bodily activity, particularly with regards to the bodily sense of balance. We found that (1) children grounded numbers both to balance states, treating positive and negative integers as symmetrically organized around 0, and to hand movement trajectories on the number line, enacting operations in a left-to-right frame of reference, and they progressively integrated these perspectives, and (2) children dynamically *re-grounded* symbolic expressions during moments of mathematical transitions and communicative difficulty. Our findings suggest that symbol grounding is dynamic and contextual, rather than primarily a progression from concrete to abstract.

Introduction

How do mathematical symbols come to have meaning for learners? This is the symbol grounding problem (Harnad, 1990), an influential question across disciplines with significant importance for the symbol-heavy domain of mathematics education. A first answer to the symbol grounding problem is that symbols take on meaning for an individual by being connected to other representations that already have meaning for them. Many scholars have argued that the ability to coordinate among different semiotic systems is a marker of, or even constitutive of, conceptual understanding (i.e., Arzarello et al., 2005; Bartollini Bussi & Mariotti, 2008; Radford, 2003). Further, instructional approaches that involve connecting representations are often recommended as a means to support students in developing conceptual understanding (e.g., Mainali, 2021, Goldin & Shteingold, 2001). Researchers have documented that linking across different symbolic representations is central in mathematics teaching, and the teachers express such links through speech and gesture (i.e., Alibali et al., 2013; Nathan, 2008; Meng et al., 2023).

However, the idea that symbolic expressions gain meaning by connecting to other representations does not explain how these a priori meaningful representations themselves became meaningful. This brings us to a second proposed answer: that symbols take on meaning for an individual by being connected to bodily experiences through perception and action: symbol grounding (Harnad, 1990). In this view, symbols experienced as meaningful "evoke particular non-symbolic cognitive resources, primarily sensory impressions" (Abrahamson, 2024). This view resonates with and in some cases has been taken up by theorists that emphasize the sensorimotor or enactive foundations of concepts (Bruner, 1966; Piaget, 1970; Varela et al., 1991), as well as math education scholars who seek to foster bodily interactions for learning purposes including concreteness fading (Fyfe et al., 2014), concrete-representational-abstract (CRA) (Agrawal & Morin, 2016), and progressive formalization (Nathan, 2012). The linking of symbols with action also shows up in the Common Core State Standards, which invite using math to model actions (NGACBP & CCSSO, 2010). If symbols must be grounded in bodily activity, this invites us to consider *what* bodily experiences might best serve this purpose.

This paper comes from a project focused on exploring underexamined bodily modalities for grounding mathematics, specifically here the bodily sense of balance. The notion of balance already plays a role in math discourse and teaching. It is often evoked *metaphorically* in the context of algebraic equations (i.e., Otten et al., 2020; Walkoe & Levin, 2020), including with tangible tools such as pan balances, buckets and beanbags (Donovan & Alibali, 2021; Stephens et al., 2022). In particular, balancing equations has been characterized as a means to leverage symmetry processing to support the *additive inverse principle*, whereby for every number x, there is a

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corresponding number, -x that sums to 0 (Varma et al., 2018). However, to date, this work does not explore grounding to the *bodily* sense of balance: experience of acceleration and orientation relative to gravity detected through the vestibular sense in our inner ear integrated with proprioceptive and visual cues. In this project, we explore: how might learning change if we ground the additive inverse principle directly to bodily experiences of balance? How might this bring meaning to symbolic expressions?

Basic neuroscience and cognition research has linked the vestibular system to cognitive development and functioning, specifically spatial and numerical processing and memory (Bigelow & Agrawal, 2015). Bodily balancing activity can impact reasoning and affective experience when considering abstract notions of balance and fairness, suggesting bodily balance experiences can conceptually ground abstract ideas (Antle et al., 2013). Despite these intriguing links, the prospective impact of vestibular stimulation on mathematical learning is just beginning to be explored (Tancredi, 2024).

In this project, we designed and investigated a pedagogical context wherein the additive inverse principle can be grounded directly to experiences of bodily balance. Our aim was to investigate the potential for vestibular-based symbol grounding, with an interest in whether and how this modality can offer sensemaking resources to children learning about negative numbers. Our primary research question is: *(How) can children come to ground symbol systems and the relations among them in the bodily sense of balance?*

The Balance Number Line: A Design for grounding the additive inverse principle in balance

In the Balance Number Line (BNL), children sit on a wooden balance board that can rock left-to-right and move their hands over a number line in front of them (Figure 1). The center of their body is aligned with point 0 on the number line, with negative numbers extending left and positive numbers extending right. As children move their hands on the number line, the shifting weight of their hands tilts the board in the direction of the hand's location. For example, placing a hand at -1 tilts the child slightly left, and placing it on 1 tilts them the same degree to the right. Placing a hand at -5 tilts them more strongly to the left. Placing one hand on -3 and the other on 3 results in a state of balance, as does placing both hands on 0. Children were invited to hold weights if desired to exaggerate this effect. The surface of the number line is magnetic, and magnetic point markers and vector arrows such as + 2 or - 5 are provided to allow children to show or plan movement sequences.

Figure 1

BNL Materials and Configuration



Children completed a series of discovery-based movement tasks presented on laminated cards. The tasks were designed to foster a balance-based conceptualization of the additive inverse principle by establishing negative numbers as the counterbalancing point for positive ones:

- In *Symmetrical Rocking*, they were invited to rock the board side to side the same amount by moving their hands on the number line. (Solution: rock to additive inverse points on each side).
- In *Maintain Balance*, they were invited to find a way to move both hands over the line while staying balanced the whole time. (Solution: move at equal distance from 0.)
- In *Compare Tilts*, they were invited to use the board to compare the absolute value of different pairs of points (i.e., |-1| & |5|). The purpose of these activities was to sensitize learners to the fact that inverse integers elicit equal tilt to each side and can counterbalance each other, and to relate the absolute value of integers to the degree of tilt elicited by the board.
- In *Arithmetic Expressions*, they were informed, "We can use numbers to write down moves, like a rocking dance. Let's try some moves from other people who have used the board." With researcher guidance, they tried enacting some arithmetic expressions such as "1+1+1+1" on the board as a series of hand movements. Vector arrows were used to scaffold planning and discussing movements as needed.
- In *Create-a-Move*, where they were informed: "Now it's your turn to come up with a move for others to try and write it down!"

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Before and after the movement tasks, children completed an assessment that consisted of (1) defining 0, negative numbers, and absolute value, (2) placing positive and negative integers onto a blank number line, and (3) solving written problems featuring negative numbers and absolute value. Four of the children had some prior knowledge of negative numbers (i.e., they answered simple arithmetic with negative integers questions correctly on a pre-screening).

In this paper, we analyze the final Create-A-Move task. We selected this task due to its heavily symbolic nature. Create-A-Move follows the *action-based embodied design* framework's approach (Abrahamson, 2014; Alberto et al., 2022) to symbol grounding, wherein symbols are introduced as means to enhance, represent, explain, or evaluate situated motor actions. The symbols introduced function as *semiotic means of objectification* (Radford, 2003) for children's perceptuomotor activity. That is, symbols bring forth a way of moving/perceiving as a thing to be talked about, and in so doing, constrain the sensorimotor activity itself through the symbol's affordances. For example in a ratio task, the introduction of a grid and numerals leads children to perceive and speak about ratio relations quantitatively, for example as a "1-per-2" relation (Abrahamson et al., 2014). Through using symbols in this way, learners are theorized to develop new perceptions of these symbols, wherein the symbols now evoke action possibilities that ground their meaning. Our task invites learners to use arithmetic symbols as a tool to communicate ways of moving. In this context, arithmetic expressions take on significance as *instructions* for how to move one's body, and these instructions must convey complete information for an imagined future reader.

Figure 2

Example Solutions to Symmetrical Rocking (Left) and Maintain Balance (Right) by 5th Grader Nia



Design iterations

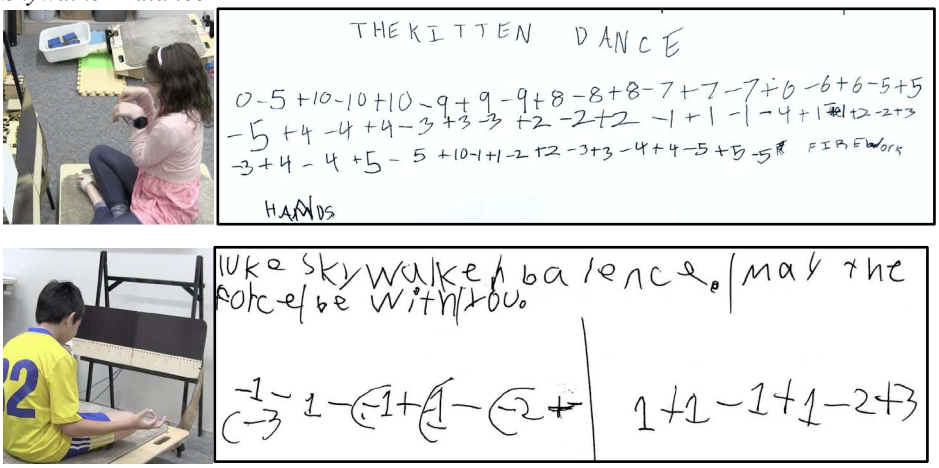
This study is situated within a broader design-based research (Cobb et al., 2003) project, Balance Board Math (Tancredi, 2024). Prior work on this project, including clinical interview and classroom studies, found that rocking-based math activities can support learning about trigonometric functions, including by cultivating new ways of perceiving graphs that draw upon balance states, evidenced in speech and gesture (i.e., Tancredi et al., 2022). In this prior work, we observed that children adapted their rocking intensity to meet their preferred sensory stimulation intensities (Tancredi et al., 2022). As such, in Create-A-Move, we aimed to ground not in prescribed vestibular actions, but rather to ways of rocking that are sensorially ergonomic and personally motivating to a given learner. In this way, our task diverges from classical action-based embodied design in that children's "Moves", while influenced by prior task solutions, are self-generated rather than the solutions to the earlier discovery-based problems.

Within the study, our initial version of Create-A-Move invited children to create a move for one of the researchers to enact on the board as a sort of role reversal to close out the session. However, we quickly observed that children spent the greatest portion of their interview time absorbed in composing and performing their moves, drawing on multiple semiotic tools and drafting often-lengthy arithmetic expressions. In the debriefing interview, children consistently highlighted that they enjoyed this activity.

Furthermore, the expressions they chose to produce directly engaged with the additive inverse principle: consider for example, the moves of rising 6th graders Cora and Tomas (Figure 3). Cora's 56-term composition, "The Kitten Dance," expresses the joy of having a pet kitten through variable patterned sweeping and alternation movements (concluding with "Firework Hands"). Tomas' "Luke Skywalker Balance" expresses the Jedi-training feeling he experienced when maintaining a state of balance on the board by drafting additive inverse expressions for each hand. The Kitten Dance involves a pattern of additive inverse terms; the Luke Skywalker Balance is two expressions that are the additive inverse of each other. Given these observations, we (1) expanded on this activity's social nature by giving children the chance to view and play other children's moves and to contribute to a growing set of moves for future participants to explore, and (2) explicitly asked children if they wanted to engage in activities we had seen with prior participants such as planning or naming their movement, sharing their move's inspiration, and performing their move themselves.

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Figure 3
Cora and Tomas Conceptualize their Compositions: the "Kitten Dance" and the "Luke Skywalker Balance"



Methods

We conducted 9 semi-structured, task-based clinical interviews (Ginsburg, 1997) (44-74 mins, median: 60) with children in grades 1-6 (1) (median: 5) exploring all BNL tasks. In this analysis, we conducted a collaborative intensive microgenetic multimodal analysis of the Create-A-Move task, drawing on grounded theory approaches (Strauss & Crobin, 1990). In videos of children's planning, composing, and performing of their arithmetic "moves," we coded every involvement of a sensorimotor modality (manual gesture, vestibular rocking) or representational media stream (number line, vector arrows, written arithmetic expressions, verbalized numbers and operations) using ELAN software. We then deductively coded children's mathematical utterances in terms of which aspect of the arithmetic expressions and/or number line they referenced (Table 2).

Table 2
Types of Quantitative Language Used When Planning Arithmetic Expressions

Focus	Description	Example
distance from zero	distance to zero, ignoring sign *(0-centered frame of reference)*	*Refer to -5 as "5"*
location	locations on the number line, including sign	*"-5"*
distance	distance between two locations, regardless of direction	*"10" for 5 to -5*
enumeration	counting out distances	*"1, 2, 3, 4"*
direction	the direction of movement *(left-to-right frame of reference)*	*"Right" "this way"*
operation	adding or subtracting *(left-to-right frame of reference)*	*"Plus 4" "minus -2"*
operand	the object of an operation	*"Negative 2"*
total	the cumulative sum or difference at the end of an expression	*"Equals 1"*

Results

Grounding in balance experiences and number line hand movements

To write an arithmetic expression, no body movement is *required*, beyond that needed for writing. However, all children enacted balance actions as part of creating their moves. In planning their arithmetic expressions, most children began by rocking their bodies. Reflecting the salient affordance of the balance board to rock symmetrically, all moves spanned negative (left tilt) and positive (right tilt) integers. For example, rising 5th grader Cecilia planned by pointing with her right hand to locations 0, 1,-1, 2, and -2 on the number line, tilting the board an increasing amount to each side. She then stated, "ok, I think I have an idea." She identified an expanding pattern of alternating additive inverse integers, moving successively further outwards from 0 symmetrically on either side. The move she went on to compose extrapolates the pattern she began here (see Figure 5). Children also grounded in rocking movements when composing their symbolic expressions. For example, Cora leaned subtly right and left as she added alternating terms in the first portion of the Kitten Dance (rocking that decreases in amplitude down to -1). Like marking practices in which dancers evoke specific aspects

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of their movement without enacting its full complexity (Kirsh, 2011), Cora schematized rocking by enacting just the direction of tilt, without the full amplitude.

Numbers as locations relative to zero-as-balance

The first symbol set that children brought to bear on their rocking movements was numbers as locations on the number line. Children's speech in early stages of the task referred to numbers as locations, often as a series of locations relative to 0 (e.g., referring to location -2 as "2"). For example, Tomas began planning his Luke Skywalker Balance composition, wherein both hands move through successive additive inverse positions, by describing each term as a distance to 0: "put your hands on 2, then 1, then 2, then 1" (Figure 4). His language highlights how he perceives these positions as symmetrical relative to 0.

Figure 4

Tomas Plans His Move in Terms of Distances From Zero



"Put your hands on 2 then 1 then 2 then 1"

Numerical operations as left-to-right trajectories

In working to compose their move as a series of arithmetic operations, children came to attend to their movement as a series of left-to-right transformations. Children grounded operations and operands in horizontal finger or hand movements along the number line. Children spoke about distances, movement direction left or right (sometimes calling it "up" or "down"), and ultimately, magnitude and direction with operations and operands. For example, as 1st grader Camila began composing her expression, she moved her finger in a unit by unit counting motion along the number line: "I want it to be um, 5, and then you have to get- wait how do you get to 5 to that many? [pointing to -3 on the number line]. I'll count: [enumerating on the number line (Figure 5)] 1, 2, 3, 4, 5, 6, 7, 8. 8, we need."

Figure 5

Camila Counting the Distance from 5 to -3



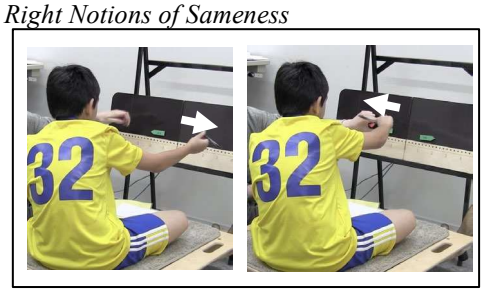
Whereas Camila first grounded the value "8" in 8 leftwards displacements of her finger, she went on to compose a repetitive sequence of "- 8 + 8" three times in her arithmetic expression. In doing so, she switched from enumerative counting of 8 units to jumping her finger directly from -4 to 4. Whereas initially "+8" was grounded in a series of 8 unit-length moves to the right, she eventually packaged it into a single "+8" rightwards vector. Across participants, many children progressively came to objectify operations as continuous swoops over the number line.

Dual-grounding: Integrating 0-centered and left-to-right frames of reference

To document their rocking moves, children needed to express rocking with a series of arithmetic operations. In doing so, children took up the number line as a site to integrate the 0-centered, symmetrical frame of reference of balance board tilts with the left-to-right trajectories denoted by arithmetic operations. Children engaged with the number line in a wide range of ways, including glancing, pointing, moving their fingers along the surface, and verbal referencing. For example, Tomas, in working on the 4th term of his expression, in which both hands move from |1| to |2|, used the number line to frame and ultimately resolve a conflict between 0-centered and operational frames (Figure 6): "And since you're back at 1, you go to 2 again. So starting here [points to 1 on right side], so you're at 1, plus 1, which is 2.[Shifts focus to left side, draws finger from -1 to -2.] I think it's minus 1- no, like to go this [points right, Figure 6A]- oh yeah, to 2 [points left, Figure 6B]."

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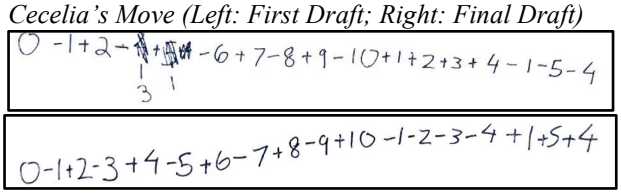
Figure 6
Tomas Points Left and Right As He Integrates Zero-Centered and Left-to-Right Notions of Sameness



Here, Tomas uses the number line to instantiate and relate two ways of thinking about similar operations: an arithmetic way (in which to do the same thing, the operation must go the same direction) and a 0-centered way (in which to do the same thing, the operation must go the opposite direction).

Learners also navigated conflict between the operational and symmetrical frames when performing moves. When rising 5th-grader Cecelia was first composing her move, she kept track of her cumulative sum with her finger. When she stopped to write the term "+2," however, she accidentally returned her finger to location 2, rather her last position, which was 1 (just before the first corrected term in Figure 7). This slippage of "+ 2" from operation to location became a sticking point when she performed her move. She began by leaning her body to each location, verbalizing and tracing with her finger on the number line for each lean, "so it goes, so you're at 0, so you go minus 1, plus 2." For + 2, she made a continuous swoop of her finger from - 1 to 1. However, here, she began saying, "minus," then paused, leaning left with her finger trailing behind. Whereas her lean anticipated arriving at location -2 in keeping with her expanding additive inverse pattern, the operation "- 4" would take her location -3. She subvocally enumerated 4 discrete unit jumps downwards, and said, "wait, I made a mistake here." The discrepancy between her 0-centered, rocking-based anticipation and her operational expression elicited a need for recoordination. She returned to enumerative counting to re-ground her sense of the operation "- 4" in stepwise finger movements on the number line to be sure it was indeed breaking her movement pattern. As these examples illustrate, learners frequently engaged in coordinative work, addressing moments of semantic misalignment as they came to dually-ground arithmetic expressions in linear horizontal number line trajectories and symmetrical 0-relative board tilt positions. Their use of language and action reflected progressive integration of a symmetry-oriented perspective with an operational perspective on the direction and span of movements.

Figure 7
Cecelia's Move (Left: First Draft; Right: Final Draft)



Re-grounding

Children did not ground every step of their equations in movement. As their compositions progressed, children often reduced their referencing of the number line. However, we observed the re-emergence of grounding movements such as tracing on the number line and tilting the board during transitions in math activity, and in moments of communication breakdown.

Re-grounding in moments of novelty or transition

Children re-grounded in movement at moments of mathematical novelty, such as changes in the pattern of their arithmetic expression. For example, as Cora composed the Kitten Dance, she re-grounded at key transition points, using movement in different ways. She composed a series of terms quietly, without body movement, but when she reached a new phase of her move which entailed movements on the left side of 0, she drew upon the number line for the first time, saying: "wait a minute [puts finger on number line at -1], how would I get from negative 1 [slides finger down to -5] to negative 5? minus 4?" (Figure 8, left). The novelty at this transition point invoked a renewed need for grounding in movement on the number line. Later, Cora reached a moment where her move

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transitioned to the positive side to repeat the pattern just done on the negative side. As she started to write this section, she leaned her body to the right, aligning herself with the space where she would be if she were enacting this part of the move (Figure 8, right).

Figure 8

Cora Re-Grounds By: Left: Moving a Finger on the Number Line, Right: Shifting her Body Right to Compose Operations on Positive Integers

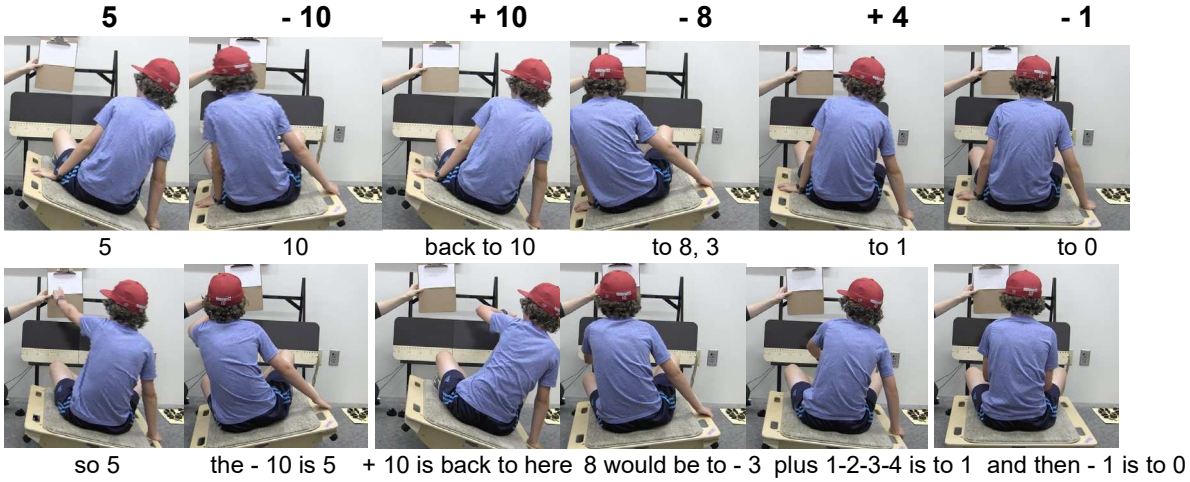


Re-grounding in moments of communicative difficulty with the researcher

Another type of catalyst for regrounding was communicative difficulty. Rising 6th grader Cal's move was inspired by a roly poly toy that starts "big" and "slows to a stop." When he first performed his move, he demonstrated the move using rocking only (Figure 9, middle), verbalizing: '5 [location], 10 [operand], back to 10 [operand] to 8 [operand], 3 [location] to 1 [location] to 0 [location]." Although his movements aligned with his written move, his speech used a mix of locations, distances, and distance to 0. The researcher, unable to follow these linguistic shifts, asked, "wait, to 8?" indicating a loss of intersubjective coordination. In response, Cal used pointing and language to explicitly connect his rocking movements with each term of his written symbolic expression and with locations on the number line (Figure 9, bottom). This time, he described his movement with operands and locations throughout: "so 5 [location], the negative 10 [operation/operand] is 5 [distance to 0], plus 10 [operation, operand] is back to here [location], 8 [operand] would be to negative 3 [location], plus 1-2-3-4 [enumeration, operation, operand] is to 1 [location], and then negative 1 [operation/operand] is to 0 [location]." In response to a communicative challenge, Cal explicitly recoordinated across semiotic systems and frames of reference.

Figure 9

Cal Performs His Move (Top) With Rocking Only (Middle) and then with Deictic Gestures (Bottom)



Discussion

Within a setting designed to ground the additive inverse principle in the bodily sense of balance, children actively grounded and re-grounded symbolic arithmetic expressions in balance and movement. To compose their arithmetic expressions, most children began by considering the rocking "move" they hoped to communicate, taking up numbers on the number line as landmark positions within a 0-centered frame of reference to express how far to tilt in either direction. This resonates with prior findings that highlighting symmetry supports integer learning (Tsang et al., 2015). As they began to compose these moves with arithmetic symbols, they used the number line to integrate this 0-centered frame of reference with the left-to-right frame of reference of arithmetic operations.

Notably, it was not the case that children achieved symbol grounding and then proceeded to limit themselves to operating within that symbol system. Instead, children *re-grounded* symbolic expressions iteratively

and responsively to cognitive or social demands. Novelty and transitions sparked re-grounding processes, as did moments of apparent conflict among different perceptions of a symbolic action such as an operation. These findings trouble the implication implicit in many extant pedagogical symbol grounding practices, such as concreteness fading, that once symbols are grounded, the concrete "stuff" can be discarded for "pure" reasoning. (Of course, even such symbol pushing has been recognized as concrete in its own ways).

In the Create-a-Move task, children were invited to produce arithmetic expressions, but they were not directly asked to move their bodies. (In fact, one child did not rock at all when planning or composing his move, and was surprised to discover that his expression summed to 0 when performed). Even so, most children engaged in bodily rocking and/or hand movements on the number line, suggesting that such activity supported their composition, either by enabling exploration of possibilities, or by enhancing accuracy of their vision. Although some of children's movements were explicitly communicative, as when asking a question about a trajectory on the number line, many of children's rocking and number line movements arose during independent work writing their compositions. For example, Cora's subtle shifts of her body to align with the side of the number line where a given section of the arithmetic expression would take place suggests an *alignment effect* (Hatzipanayioti & Avraamides, 2021), wherein alignment of frames between her symbolic and bodily activity may have facilitated her symbol work.

This task exemplifies the embodied design genre (Abrahamson, 2014) in that children moved in a new way and then used symbols to document and communicate about this new way of moving, grounding the meaning of symbol systems in their physical movement in the process. We observed that such procedures highly motivated children to engage in complex semiotic coordination. We extend prior work within this genre by inviting children to create movements meaningful to them, and by expanding the modalities of engagement. Mathematics instruction follows a notably oculocentric tradition (Abrahamson et al., 2019), emphasizing visual experiences and representations over other modalities. By embracing children's vestibular engagements as the grounding for symbolic activity, vestibular-centric bodily practices such as neurodivergent stimming are legitimized and welcomed into mathematical discourse (Tancredi & Abrahamson, 2024). This design offers an early proof-of-concept for a design genre that broadens sensory engagements in service of epistemological pluralism (Turkle & Papert, 1991).

Our findings suggest that the meanings of symbolic expressions can be grounded to vestibular experiences. A next step that would clarify the nature and potential of vestibular grounding would be to experimentally test the impact of the balance aspect of the design compared to a number line only configuration. How do number line and balance movements, separately and in combination, influence engagement, learning, transfer, and retention, relative to more traditional instructional practices? In terms of next steps for making Create-a-Move an effective pedagogical activity, our findings suggest the importance of opportunities for semiotic and social coordination. To this end, we propose to (1) amplify multimodal feedback so that learners can hear and see alignment between their planned and performed compositions by adding sensor-driven light and sound elements to the number line, and (2) expand the motivating context of social sharing by establishing a partnership with a museum space to develop an interactive exhibit for composing math expressions and performing expressions written by others.

Conclusion

Through the Create-A-Move task, children were motivated to make use of math symbolic expressions to communicate personally meaningful movement patterns. The task and materials created a field of promoted action wherein children dual-grounded arithmetic expressions in both movements on the number line and tilts on the balance board. Whereas the symmetrical nature of the balance board foregrounded attending to distances and locations relative to 0, highlighting additive inverse property and absolute value, the symbolic arithmetic expressions foregrounded operations with specific directionality left-to-right. Given these dual frames of reference, children were motivated to take up the number line as a tool for integration of left-to-right and 0-centered frames. Notably, children's grounding actions explicitly mapping symbols onto movements increased during moments of cognitive or communicative complexity, suggesting that symbol grounding may not be singular achievement, but rather a dynamic participant in ongoing thought.

Endnotes

(1) The first 3 participants completed two sets of activities, those described here and a second set focusing on equations and inequalities. After this first stage, participants were restricted to grades 4-6 and the full interview was allotted to the number line focused tasks. Demographics (parent-reported, not mutually exclusive): 4 female, 5 male; 2 black/african american, 1 asian, 7 white/caucasian; 3 latino; 2 neurodivergent (autistic/ADHD and ADHD/dyslexia/dyscalculia).

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Acknowledgments

The Balance Number Line was first developed at the Embodied Design Research Forum at UC Berkeley. We warmly thank Dr. Dor Abrahamson for his formative input on this initial design, undergraduate researcher Katie McCaskey for her work input on task design and participation in early data collection and transcription work, our participants for their curiosity, creativity, and openness, and the Cognitive Development and Cognition Lab for their comments on these data.