Beyond *'Solve for x'*: Integrating Equations with Conceptual Understanding

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Abstract: This research attempts to describe how biology students' integrate equations with disciplinary concepts. First, I assess how students' explanations of biological phenomena change after solving word problems and receiving feedback from a quantitative simulation. Then, I describe a novel learning environment for further exploring design principles that support students in integrating equations with conceptual knowledge. With biology careers increasingly demanding mathematical approaches, students are better prepared to enter these careers if they can integrate equations with conceptual knowledge.

Introduction

Historically, scientists have touted mathematics as *the language of nature*. Modern Learning Sciences perspectives adopt a less romantic approach by arguing that mathematics supports communication (Lemke, 2003) and cognition (Schwartz & Moore, 1998). Science students who receive instruction with mathematics, however, often fail to integrate equations and algorithms with conceptual knowledge. The present research aims to assess how science students' explanations of phenomena change after solving word problems and receiving feedback from a quantitative simulation. The research further aims to identify design principles that support learning with mathematics in science. Integrating mathematical representation and procedure with conceptual knowledge better prepares students for research careers and collaborations across STEM disciplines.

Equations in Science Education

Regarding instruction, physics and chemistry educators traditionally teach science with equations and assess students' understandings by posing word problems. Educational research in these disciplines demonstrates that students struggle to integrate equations and algorithms with conceptual understandings of science content (e.g. Nakhleh, 1993). When educators attempt to relieve difficulty, improve performance, or teach non-science majors, they often shift to a conceptual approach that eliminates equations from instruction. Treating mathematical and conceptual approaches as separate, however, reflects a false dichotomy. The present research aims to improve our knowledge of how students integrate equations with disciplinary concepts when learning with technologies designed to foster integration.

The Knowledge in Pieces Approach

The KiP framework offers a systematic approach to analyzing how students' organize *knowledge elements* vis-à-vis equations. KiP refers to the fragmented bits of knowledge that students coordinate to complete disciplinary tasks (diSessa, 1993). The KiP approach maintains that students' knowledge elements exists as a *system* that resides in a state of flux as opposed to a coherent structure organized around stable theories and beliefs.

The KiP framework was applied previously to characterize how students understand physics equations (Sherin, 2001). By Sherin's analysis, students understand physics equations by using the knowledge elements *symbolic forms*. All forms consist of a *symbolic template* and a *conceptual schema*. The template denotes the abstract structure that students recognize in an equation and the schema denotes the general idea that the template represents. Students interpret equations by recognizing their abstract structure. For instance, upon recognizing a term as a coefficient, students explain that the coefficient "just tunes" the size of an effect. The present research aims to (1) explore the generativity of Sherin's framework by extending it from physics to biology education and (2) identify domain-specific design principles that support biology students in integrating equations with conceptual knowledge.

Methods

Students (n = 10; M = 20 years; *Range* = 20-23) were concurrently enrolled in a (200-level) biology course titled *Homeostasis*. Students participated in semi-structured interviews that asked them to answer the question, "How does a cell *generate* a resting membrane potential?" Then, students were randomly assigned to either solve word problems with a quantitative simulation that mathematically models how cells generate resting membrane potentials or, for comparison, to observe a narrated animation that explained this process. After receiving instruction, students answered the question once more. Students' explanations were video and audio recorded for later transcription and analysis. Data were collected in a laboratory setting.

Regarding the domain under consideration, biological cells possess the capacity to generate trans-membrane voltages. If an ionic species is more concentrated in the intracellular fluid than in the extracellular fluid, then the species will tend to diffuse down the chemical gradient due to random motions. Because cells possess ion-selective protein channels, some ions will diffuse down their gradient, carrying their charge with them. This causes oppositely charged particles to separate at the membrane. The value at which the electrical driving forces balance the chemical driving forces is the cell's resting membrane potential or steady state voltage. We can model this system mathematically with the Nernst and Goldman equations (see Equation 1).

$$(1)\ E_x = 60\ \cdot log\frac{[X]_o}{[X]_i}\ (2)\ V_m = \frac{G_x}{G_T}\cdot E_x + \frac{G_y}{G_T}\cdot E_y + \frac{G_z}{G_T}\cdot E_z \ldots$$

Equation 1. (1) Nernst potential. (2) Goldman equation transformed into the Nernst potential ran in series. Coefficients denote the relative conductance for each ionic species.

Using techniques borrowed from diSessa (1993) and Sherin (2001), students' linguistic descriptions were analyzed and categorized according to knowledge elements.

Results

Preliminary analysis suggests that students rely upon the symbolic form cluster *competing terms*. Competition refers to two or more influences that vie in a struggle to produce some final effect. Within this cluster, students draw heavily upon the symbolic form *balance*. Sherin (2001) describes *balance* as "two influences, each associated with a side of the equation, in balance so that the system is in equilibrium" (pg. 533). Similarly, diSessa's (1993) describes the knowledge element *dynamic balance* as "a pair of forces or directed influences that are in conflict and happen to balance each other" (pg. 222). For instance, one student stated, "So there has to be an equal amount of charge, not an equal amount of charge but it's kind of like two opposing forces. Like the concentration and also the charge."

Whereas instruction with a quantitative simulation promoted students to shift to cuing the *equilibrium* (stability within parameters) and *equilibration* (return to stability) knowledge elements, instruction with a narrated animation promoted students to shift to the *balance* element. For instance, after solving word problems with the simulation, one student stated, "When things get out of whack […] the cell wants to get things back to its composition of the way that it works most efficiently. So it's going to either take the ions out of the cell or take the ions from the outside of the cell and bring them in to reach equilibrium." Thus, she cues the *equilibration* element. In contrast, a student who experienced the animation stated, "There comes a point where they reach a balance, where the diffusion force will equal the electric force." Thus, he cues the *balance* element.

Conclusions and Future Directions

The KiP framework holds promise for knowledge analysis in biology. The initial phase of the investigation suggests that learning with equations *can* shift the cuing priority of knowledge elements differently than learning without equations. Unfortunately, these results suggest that learning with equations in the absence of explicit instructional support fails to shift students towards productive mechanistic reasoning compared to learning with a narrated animation. This marks the first step towards the next phase of the investigation where the goal will be to design a learning environment that helps students better integrate equations with disciplinary concepts. Specifically, the design will aim to leverage the affordances of both the quantitative simulation and the narrated animation. The potential design solution aims to foster reflection and the integration of knowledge by providing students the opportunity to relate the two instructional media to each other and to their own understanding. The design aims to accomplish these ends by promoting students to generate representations of their knowledge while learning with equations.

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